

Key Notes

CHAPTER – 2 POLYNOMIALS

1. Polynomials in one Variable
2. Zeroes of a Polynomial
3. Remainder Theorem
4. Factorisation of Polynomials
5. Algebraic Identities

- **Constants:** A symbol having a fixed numerical value is called a constant.
- **Variables:** A symbol which may be assigned different numerical values is known as variable.
- **Algebraic expressions:** A combination of constants and variables. Connected by some or all of the operations +, -, X and is known as algebraic expression.
- **Terms:** The several parts of an algebraic expression separated by '+' or '-' operations are called the terms of the expression.
- **Polynomials:** An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

(i) $5x^2 - 4x^2 - 6x - 3$ is a polynomial in variable x.

(ii) (ii) $5 + 8x^{\frac{3}{2}} + 4x^{-2}$ is an expression but not a polynomial.

Polynomials are denoted by p(x), q(x) and r(x) etc.

- **Coefficients:** In the polynomial $x^3 + 3x^2 + 3x + 1$, coefficient of x^3 , x^2 , x are 1, 3, 3 respectively and we also say that +1 is the constant term in it.
- **Degree of a polynomial in one variable:** In case of a polynomial in one variable the highest power of the variable is called the degree of the polynomial.
- **Classification of polynomials on the basis of degree.**

Degree	Polynomial	Example
(a) 1	Linear	$x + 1$, $2x + 3$ etc.
(b) 2	Quadratic	$ax^2 + bx + c$ etc.
(c) 3	Cubic	$x^3 + 3x^2 + 1$ etc. etc.
(d) 4	Biquadratic	$x^4 - 1$

Classification of polynomials on the basis of no. of terms

Key Notes

No. of terms	Polynomial & Examples.
(i) 1	Monomial - $5, 3x, \frac{1}{3}y$ etc.
(ii) 2	Binomial - $(3+6x), (x-5y)$ etc.
(iii) 3	Trinomial- $2x^2+4x+2$ etc. etc.

- **Constant polynomial:** A polynomial containing one term only, consisting a constant term is called a constant polynomial the degree of non-zero constant polynomial is zero.
- **Zero polynomial:** A polynomial consisting of one term, namely zero only is called a zero polynomial. The degree of zero polynomial is not defined.
- **Zeroes of a polynomial:** Let $p(x)$ be a polynomial. If $p(\alpha)=0$, then we say that is a zero of the polynomial of $p(x)$.
- **Remark:** Finding the zeroes of polynomial $p(x)$ means solving the equation $p(x)=0$.
- **Remainder theorem:** Let $f(x)$ be a polynomial of degree $n \geq 1$ and let a be any real number. When $f(x)$ is divided by $(x-a)$ then the remainder is $f(a)$
- **Factor theorem:** Let $f(x)$ be a polynomial of degree $n > 1$ and let a be any real number.
 - (i) If $f(a) = 0$ then $(x-a)$ is factor of $f(x)$
 - (ii) If $(x-a)$ is factor of $f(x)$ then $f(a) = 0$
- **Factor:** A polynomial $p(x)$ is called factor of $q(x)$ divides $q(x)$ exactly.
- **Factorization:** To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Some algebraic identities useful in factorization:

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x-y)(x+y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = 3xyz$ if $x+y+z = 0$
