#### Quadrilaterals <u>Exercise – 8.1</u>

Write the correct answer in each of the following:

### 1. Three angles of a quadrilateral are 75°, 90° and 75°. The fourth angle is

- (A) 90º
- (B) 95º
- (C) 105º
- (D) 120º
- **Sol.** Fourth angel of the quadrilateral

$$= 360^{\circ} - (75^{\circ} + 90^{\circ} + 75^{\circ})$$

- = 360° 240°
- = 120°

Hence, (d) is the correct answer.

## 2. A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is

- (A) 55º
- (B) 50º
- (C) 40º
- (D) 25º
- **Sol.** ABCD is a rectangle in which diagonal AC is inclined to one side AB of the rectangle at an angle of 25°.



Now, AC = BD

[:: Diagonal of a rectangle are equal]

$$\therefore \qquad \frac{1}{2}AC = \frac{1}{2}BD$$

$$\Rightarrow 0A = 0B$$

In  $\triangle AOB$ , we have OA = OB

 $\therefore \qquad \angle OBA = \angle OAB = 25^{\circ}$ 

$$\therefore \qquad \angle AOB = 180^{\circ} - (25^{\circ} + 25^{\circ}) = 130^{\circ}$$

 $\angle AOB$  and  $\angle AOD$  from angle of a linear pair.

 $\therefore \angle AOB + \angle AOD = 180^{\circ}$ 

 $\Rightarrow \angle AOD = 180^\circ - 130^\circ = 50^\circ$ 

Hence, the acute angel between the diagonal is 50°. Therefore, (b) is the correct answer.

#### 3. ABCD is a rhombus such that $\angle ACB = 40^{\circ}$ . Then $\angle ADB$ is

- (A) 40º
- (B) 45º
- (C) 50°
- (D) 60º
- **Sol.** ABCD is a rhombus such that  $\angle ACB = 40^{\circ}$ .

We know that diagonals of rhombus bisect each other at right angles. In right  $\Delta BOC$ , we have



$$\angle OBC = 180^{\circ} - (\angle BOC + \angle BCO)$$
$$= 180^{\circ} - (90^{\circ} + 40^{\circ}) = 50^{\circ}$$
$$\therefore \angle DBC = \angle OBC = 50^{\circ}$$
Now

Now,

$$\angle ADB = \angle DBC \qquad [Alt. int. \angle s]$$
  
$$\therefore \angle ADB = 50^{\circ} \qquad [\therefore \angle DBC = 50^{\circ}]$$

Hence, (c) is the correct answer.

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if

(A) PQRS is a rectangle

- (B) PQRS is a parallelogram
- (C) diagonals of PQRS are perpendicular

(D) diagonals of PQRS are equal.

- **Sol.** If diagonals of PQRS are perpendicular. Hence, (c) is the correct answer.
- 5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if
  - (A) PQRS is a rhombus
  - (B) PQRS is a parallelogram
  - (C) diagonals of PQRS are perpendicular
  - (D) diagonals of PQRS are equal.

**Sol.** If diagonals of PQRS are equal. Hence, (d) is the correct answer.

## 6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a

- (A) rhombus
- (B) parallelogram
- (C) trapezium
- (D) kite
- **Sol.** As angle A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4, so let the angles A, B, C and D be 3x, 7x, 6x and 4x.

Now, sum of the angle of a quadrilateral is 360°.

 $3x + 7x + 6x + 4x = 360^{\circ}$ 

 $\Rightarrow 20x = 360^{\circ} \Rightarrow x = 360^{\circ} \div 20 = 18^{\circ}$ 

So, the angles A, B, C and D of quadrilateral ABCD are  $3 \times 18^{\circ}$ ,  $7 \times 18^{\circ}$ ,  $6 \times 18^{\circ}$  and  $4 \times 18^{\circ}$  i.e., 54°, 126°, 180° and 72°

Now, AD and BC are two lines which are cut by a transversal CD such that the sum of angles  $\angle C$  and  $\angle D$  on the same side of transversal is  $\angle C + \angle D = 180^{\circ} + 72^{\circ} = 180^{\circ}$  $\therefore AD \parallel BC$ 

So, ABCD is a quadrilateral in which one pair of opposite sides are parallel. Hence, ABCD is a trapezium.

Hence, (c) is the correct answer.

- 7. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a
  - (A) rectangle
  - (B) rhombus
  - (C) parallelogram
  - (D) quadrilateral whose opposite angles are supplementary
- **Sol.** PQRS is a quadrilateral whose opposite angles are supplementary.



Hence, (d) is the correct answer.

- 8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form
  - (A) a square
  - (B) a rhombus
  - (C) a rectangle

- (D) any other parallelogram
- **Sol.** PNQM is a rectangle.



Hence, (C) is the correct answer.

- 9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is
  - (A) a rhombus
  - (B) a rectangle
  - (C) a square
  - (D) any parallelogram
- **Sol.** The figure will be a rectangle. Hence, (b) is the correct answer.
- 10. D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is
  - (A) a square
  - (B) a rectangle
  - (C) a rhombus
  - (D) a parallelogram
- **Sol.** Since the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it, so



 $\therefore \qquad DE = \frac{1}{2}BC \text{ and } DE ||BC$ 

Similarly,  $DP = \frac{1}{2}AO$  and DP||AO

And

 $EQ = \frac{1}{2}AO$  and EQ||AO

 $\therefore DP = EQ$ 

[: Each =  $\frac{1}{2}AO$ ]

And DP||EQ [∵ DP||AO and EQ||AO]

Now, DEOQ is quadrilateral in which one pair of its opposite sides is equal and parallel. Therefore, quadrilateral DEQP is a parallelogram. Hence, (d) is the correct answer.

### 11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

(A) ABCD is a rhombus

(B) diagonals of ABCD are equal

(C) diagonals of ABCD are equal and perpendicular

(D) diagonals of ABCD are perpendicular

**Sol.** If diagonal of ABCD are equal and perpendicular. Hence, (c) is the correct answer.

# 12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point 0. If $\angle DAC = 32^{\circ}$ and $\angle AOB = 70^{\circ}$ , then $\angle DBC$ is equal to

- (A) 24º
- (B) 86º
- (C) 38º
- (D) 32º
- **Sol.** AD is a parallel to BC and AC cuts them,



$$\angle DAC = \angle ACB$$
 [Alt. int.  $\angle s$ ]  
 $\angle DAC = 32^{\circ}$ 

$$\therefore \qquad \angle ACB = 32^{\circ}$$

Now, in 
$$\triangle BOC$$
 , CO is produced to A

 $\therefore \qquad \text{Ext. } \angle BOA = \angle OCB + \angle OBC \qquad [By exterior angle theorem]$ 

$$\Rightarrow$$
 70° = 32° +  $\angle OBC$ 

$$\therefore \qquad \angle OBC = 70^\circ - 32^\circ = 38^\circ$$

Hence,  $\angle DBC = 38^{\circ}$ .

Therefore, (C) is the correct answer.

#### 13. Which of the following is not true for a parallelogram?

- (A) opposite sides are equal
- (B) opposite angles are equal
- (C) opposite angles are bisected by the diagonals
- (D) diagonals bisect each other.
- **Sol.** Opposite angles are bisected by the diagonals. This is not true for a parallelogram. Hence, (c) is the correct answer.
- 14. D and E are the mid-points of the sides AB and AC respectively of Δ ABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is
  - (A)  $\angle DAE = \angle EFC$ (B) AE = EF(C) DE = EF
  - (D)  $\angle ADE = \angle ECF$ .
- **Sol.** We need DE = EF.



Hence, (c) is the correct answer.

#### Quadrilaterals <u>Exercise 8.2</u>

- 1. Diagonals AC and BD of a parallelogram ABCD intersect each other at 0. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.
- **Sol.** We know that diagonals of a parallelogram bisect each other.

 $\therefore \qquad AC = 2 \times OA = 2 \times 3 \text{ cm} = 6 \text{ cm}$ 

And  $BD = 20D = 2 \times 2 \text{ cm} = 4 \text{ cm}$ 

Hence, lengths of AC and BD are 6 cm and 4 cm respectively.

- 2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.
- **Sol.** This statement not true. Diagonals of a parallelogram bisect each other.
- 3. Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral? Why or why not?
- **Sol.** Sum of these angles  $110^\circ + 80^\circ + 70^\circ + 95^\circ = 355^\circ$ But, sum of the angles of a quadrilateral is always 360°. Hence,  $110^\circ$ ,  $80^\circ$ ,  $70^\circ$  and  $95^\circ$ cannot be the angles of a quadrilateral.
- 4. In quadrilateral ABCD,  $\angle A + \angle D = 180^{\circ}$ . What special name can be given to this quadrilateral?
- **Sol.** In quadrilateral ABCD,  $\angle A + \angle D = 180^{\circ}$  i.e., the sum of two consecutive angles is 180°. So, pair of opposite side AB and CD are parallel. Therefore, quadrilateral ABCD is trapezium. Hence, special name which can be given to this quadrilateral ABCD is trapezium.
- 5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?
- **Sol.** All the angle of a quadrilateral are equal. Also, the sum of angles of a quadrilateral is 360°. Therefore, each angle of quadrilateral is 90°. So, the given quadrilateral is a rectangle. Hence, special name given quadrilateral is rectangle.
- 6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.
- **Sol.** The given statement is not true. Diagonals of a rectangle need not to be perpendicular.
- 7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.
- **Sol.** No, because then the sum of four angles of the quadrilateral will be more than 360° whereas sum of four angles of a quadrilateral is always equal to 360°.
- 8. In  $\triangle$  ABC, AB = 5 cm, BC = 8 cm and CA= 7 cm. If D and E are respectively the midpoints of AB and BC, determine the length of DE.

**Sol.** In  $\triangle$  ABC, AB = 5 cm, BC = 8 cm and CA= 7 cm. D and E are respectively the mid-points of AB and BC.

$$\therefore DE = \frac{1}{2}AC = \frac{1}{2} \times 7cm = 3.5cm$$
 [Using the mid-point theorem]

9. In Fig.8.1, it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or why not?



**10.** In Fig.8.2, ABCD and AEFG are two parallelograms. If  $\angle C = 55^{\circ}$ , determine  $\angle F$ .



**Sol.** We know that opposite angle of parallelogram are equal. In parallelogram ABCD, we have

> $\angle A = \angle C$ But,  $\angle C = 55^{\circ}$  [Given]  $\therefore \quad \angle A = 55^{\circ}$ Now, in parallelogram AEFG, we have  $\angle F = \angle A = 55^{\circ}$ Hence,  $\angle F = 55^{\circ}$ .

#### 11. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.

**Sol.** We know that an acute angle is less than 90°. All the angles of a quadrilateral cannot be acute angles because than equal sum of a quadrilateral will be than a 360°, whereas angle sum of quadrilateral is 360°.

#### 12. Can all the angles of a quadrilateral be right angles? Give reason for your answer.

**Sol.** Yes, all the angles of a quadrilateral can be right angles. Angles sum of a quadrilateral will be 360°, which is true.

#### 13. Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35^{\circ}$ , determine $\angle B$ .

- **Sol.** As the diagonals of a quadrilateral ABCD bisect each other, so ABCD is a parallelogram. Now, ABCD is a parallelogram
  - $\therefore \qquad \angle A + \angle B = 180^{\circ}$

#### [:: Adjacent angles of a parallelogram are supplementary]

- $\therefore \qquad 35^\circ + \angle B = 180^\circ$
- $\Rightarrow \qquad \angle B = 180^{\circ} 35^{\circ} = 145^{\circ}$

#### 14. Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, determine CD.

**Sol.** Since opposite angles of a quadrilateral ACBD are equal. If AB = cm, determine CD.

[ :: Opposite sides of a parallelogram are equal]

But, AB = 4cm, therefore CD = 4cm. Hence, CD = 4cm.

#### Quadrilaterals Exercise 8.3

- 1. One angle of a quadrilateral is of 108° and the remaining three angles are equal. Find each of the three equal angles.
- **Sol.** One angle of a quadrilateral is of 180° and let each of the three remaining equal angles be  $x^{0}$ .

As the sum of the angles of a quadrilateral is 360°.

 $\therefore 180^{\circ} + x + x + x = 360^{\circ} \Longrightarrow 3x = 360^{\circ} - 180^{\circ} = 252^{\circ}$ 

 $\therefore \qquad x = 252^\circ \div 3 = 84^\circ$ 

Hence, each of the three angles be 84°.

- 2. ABCD is a trapezium in which AB || DC and  $\angle A = \angle B = 45^{\circ}$ . Find angles C and D of the trapezium.
- **Sol.** ABCD is a trapezium in which AB||DC.



Now, AB||DC and AD is transversal.

$$\therefore \qquad \angle A + \angle D = 180^{\circ}$$
[:: Sum of interior angles on the side of the transversal is 180°]  

$$\Rightarrow \qquad 45^{\circ} + \angle D = 180^{\circ}$$

$$\Rightarrow \qquad \angle D = 180^{\circ} - 45^{\circ} = 135^{\circ}$$
Similarly,  $\qquad \angle B + \angle C = 180^{\circ}$   

$$\Rightarrow \qquad 45^{\circ} + \angle C = 180^{\circ}$$

$$\Rightarrow \qquad \angle C = 180^{\circ} - 45^{\circ} = 135^{\circ}$$
Hence,  $\angle A = \angle B = 45^{\circ}$  and  $\angle C = \angle D = 135^{\circ}$ .

- 3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.
- **Sol.** In quadrilateral PBQD,



 $\Rightarrow 60^{\circ} + 90^{\circ} + \angle B + 90^{\circ} = 360^{\circ}$  $\angle B + 240^\circ = 360^\circ$  $\Rightarrow$  $\angle B = 360^{\circ} - 240^{\circ}$  $\Rightarrow$  $\angle B = 120^{\circ}$  $\Rightarrow$ Now,  $\angle ADC = \angle B = 120^{\circ}$ [:: Opposite angles of a parallelogram are equal]  $\angle A + \angle B = 180^{\circ}$ [:: Sum of consecutive interior angles is 180°]  $\angle A + 120^{\circ} = 180^{\circ}$  $\Rightarrow$  $\angle A = 180^{\circ} - 120^{\circ}$  $\Rightarrow$  $\angle A = 60^{\circ}$  $\Rightarrow$  $\angle C = \angle A = 60^\circ$ But, [:: Opposite angles of a parallelogram are equal]

- 4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.
- **Sol.** In  $\triangle APD$  and  $\triangle BPD$ , we have

$$AP = BP$$

$$Common side]$$
So, by SAS Criterion of congruence, we have  

$$AAPD \cong \Delta BPD$$

$$AAPD \cong \Delta BPD$$

$$AA = 23$$

$$CPCT]$$
But, 
$$23 = 24$$

$$CPCT]$$

$$A = 23 = 24$$

$$CPCT]$$
But, 
$$23 = 24$$

$$CPCT]$$

$$A = 23 = 24$$

$$CPCT]$$

$$A = 23 = 24$$

$$CPCT]$$

$$CPCTCT$$

$$C$$

 $=120^{\circ}$  [:: Opposite angles of a rhombus are equal]

 $\therefore$   $\angle ADC = \angle ABC = 120^{\circ}$  [Same reason as above]

- 5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.
- **Sol.** Given: A parallelogram ABCD: E and F are points of diagonal AC of parallelogram ABCD such that AE = CF.



To prove: BFDE is parallelogram. Proof: ABCD is s parallelogram.

 $\therefore OD = OB \qquad ...(1) [ \because Diagonals of parallelogram bisect each other]$  $OA = OC \qquad ...(2) [Same reason as above]$  $AE = CF \qquad ...(3) [Given]$ Subtracting (3) from (2), we getOA - AE = OC - CF $<math display="block">\Rightarrow OE = OF \qquad ...(4)$  $\therefore BFDE is parallelogram. [ \because OD = OB and OE = OF]$ Hence, proved.

- 6. E is the mid-point of the side AD of the trapezium ABCD with AB||DC. A line through E drawn parallel to AB intersect BC at F . Show that F is the mid-point of BC. [Hint: Join AC]
- **Sol.** Given: A trapezium ABCD in which AB||CD and E is mid-point of the side AB. Also, EF||AB. To prove: F is the mid-point of BC.



Construction: Join AC which intersect EF at O. Proof: In  $\triangle ADC$ , E is the mid-point of AD and EF||DC.

 $[:: EF||AB \text{ and } DC||AB \Rightarrow AB||EF||DC]$ 

∴ 0 is the mid-point of AC. [Converse of mid – point theorem] Now, in  $\triangle CAB$ , 0 is the mid-point of AC and OF||AB.  $\Rightarrow$  0F bisects BC.

Or F is the mid-point of BC. Hence, proved. 7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a  $\triangle$  ABC as shown in Fig.8.5. Show that  $BC = \frac{1}{2}QR$ .



**Sol.** Given: Triangle ABC and PQR in which AB||PQ, BC||RQ and CA||PR.

To prove:  $BC = \frac{1}{2}QR$ 

Proof: Quadrilateral RBCA is a parallelogram.

[:: RA||BC and BR||CA][:: Opposite side of parallelogram] RA = BC...(1) *.*.. Now, quadrilateral BCQA is a parallelogram. AQ = BC[:: Opposite side of parallelogram] ...(2) *.*.. Adding (1) and (2), we get RA + AQ = 2BCQR = 2BC $\Rightarrow$  $BC = \frac{1}{2}QR$  $\Rightarrow$ Hence, proved.

- 8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that  $\Delta$  DEF is also an equilateral triangle.
- **Sol.** Given:  $\Delta$  ABC is an equilateral triangle. D, E and F are the mid-points of the sides BC, CA and AB, respectively of  $\Delta$  ABC.



To prove:  $\Delta$  DEF is an equilateral triangle. Proof: EF joints mid-points of sides of AB and AC respectively.  $\therefore EF = \frac{1}{2}BC \qquad ...(1) [Mid-point theorem]$ Similarly,  $DE = \frac{1}{2}BC \qquad ...(2) [Mid-point theorem]$  $DF = \frac{1}{2}AC \qquad ...(3) [Mid-point theorem]$ But, AB = BC = CA ...(4) [Sides of an equilateral  $\Delta$  ABC] From (1), (2), (3) and (4), we have DE = EF = FD $\therefore \Delta DEF$  is an equilateral triangle. Hence, proved.

- 9. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP= CQ (Fig. 8.6). Show that AC and PQ bisect each other.
- **Sol.** Points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ.



10. In Fig. 8.7, P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.

**Sol.** 
$$\angle BAP = \angle DAP = \frac{1}{2} \angle A$$
 ...(1)



Since ABCD is a parallelogram, we have

 $\angle A + \angle B = 180^{\circ} \qquad \dots (2)$ 

[:: Sum of interior angles on the same sides of transversal is 180°] In  $\Delta ABP$ , we have

$$\angle BAP + \angle B + \angle APB = 180^{\circ}$$
  

$$\Rightarrow \frac{1}{2} \theta \angle A = 180^{\circ} - \angle A + \angle APB = 180^{\circ}$$
 [Using (1) and (2)]  

$$\Rightarrow \angle APB = \frac{1}{2} \angle A$$
 ...(3)  
From (1) and (2) we get

From (1) and (3), we get  $\angle BAP = \angle APB$ 

$$BP = AB \qquad ...(4)$$

[:: Side of opposite to equal angles are equal]

Since opposite sides of a parallelogram are equal, we have

$$AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$$
  
$$\Rightarrow \quad \frac{1}{2}AD = BP \qquad [\because P \text{ is the mid-point of BC}]$$
  
$$\Rightarrow \quad \frac{1}{2}AD = AB \qquad [\because From (4), BP = AB]$$

Since, opposite sides of a parallelogram are equal, we have

$$\frac{1}{2}AD = CD \Longrightarrow AD = 2CD$$

Hence, proved.

#### Quadrilaterals <u>Exercise 8.4</u>

- 1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.
- **Sol.** ABC is an isosceles right triangle with AB = BC. A square BFED is inscribed in triangle ABC so that  $\angle B$  = common = 90°.



- 2. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF
- **Sol.** ABCD is a parallelogram, in which AB = 10cm and AD = 6cm. The bisector of  $\angle A$  meets DC in E. AE and BC produced meet at F.



	$\angle EAD = \angle EFB$	(2) [ Alt. ∠s ]
$\Rightarrow$	$\angle BAE = \angle EFB$	[From (1) and (2)]
<i>.</i> •.	BF = AB	[ $\because$ Sides opposite to equal $\angle s$ are equal]
$\Rightarrow$	BF = 10 cm	[∵ AB = 10 cm]
$\Rightarrow$	BC + CF = 10 cm $\Rightarrow$	6  cm + CF = 10  cm
		[:: BC = AD = 6 cm, opposite sides of a   gm]
$\Rightarrow$	CF = 10 - 6 cm = 4	cm

- 3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.
- **Sol.** Given: A quadrilateral ABCD in which AC = BD and P, Q, R and S are respectively the midpoint of the sides of AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a rhombus.

Proof: In  $\triangle ABC$ , P and Q are the mid-points of AB and BC respectively.

That is, PQ joins mid-points of sides AB and BC.

 $PQ \parallel AC$ *.*•. ...(1)  $PQ = \frac{1}{2}AC$ ...(2) [Mid-point theorem] And In  $\Delta ADC$ , R and S are the mid-points of CD and AD respectively.  $SR \parallel AC$ *.*.. ...(3)  $SR = \frac{1}{2}AC$ ...(4) [Mid-point theorem] And From (1) and (3), we get PQ||SR From (2) and (4), we get PQ = RS $\Rightarrow$  PQRS is a parallelogram. In  $\Delta DAB$ , SP joins mid-points of sides of DA and AB respectively.  $SP = \frac{1}{2}BD$ ...(5) [Mid-point theorem] *:*.. AC = BD...(6) [Given] From equations (2), (5) and (6), we get SP = PQParallelogram PQRS is a rhombus. ... Hence, proved.

- 4. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC *L*BD. Prove that PQRS is a rectangle.
- **Sol.** Given: A quadrilateral ABCD in which  $AC \perp BD$  and P, Q, R and S are respectively the mid-points of the sides AB, BC CD and DA of quadrilateral ABCD.



To Prove: In  $\triangle ABC$ , P and Q are the mid-point od sides AB and BC respectively. That is, PQ joins mid-points of sides AB and BC.

 $PQ \parallel AC$ *.*.. ...(1)  $PQ = \frac{1}{2}AC$ ...(2) [Mid-point theorem] And In  $\triangle ADC$ , R and S are the mid-points of CD and AD respectively.  $SR \parallel AC$ ...(3) *:*..  $SR = \frac{1}{2}AC$ ...(4) [Mid-point theorem] And From (1) and (3), we get PQ||SR From (2) and (4), we get PO = RS $\Rightarrow$  PQRS is a parallelogram. PQ||AC [Proved above] PG||GF  $\Rightarrow$ In  $\triangle ABD$ , PS joins mid-points of sides AB and AD respectively. PS||MD [Mid-point theorem] *.*.. PG||EF  $\Rightarrow$  $\Rightarrow$  PEFG is a parallelogram [:: PE||GF and PG||EF] [:: Opposite angles of a parallelogram are equal]  $\Rightarrow$  $\angle 1 = \angle 2$  $\angle 1 = 90^{\circ}$  $[:: AC \perp BD]$ But.  $2 = 90^{\circ}$ *.*.. Parallelogram PQRS is a rectangle.  $\Rightarrow$ 

- 5. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC⊥BD. Prove that PQRS is a square.
- **Sol.** Given: A quadrilateral ABCD is which AC = BD and AC⊥BD. P, Q, R and S respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a square. Proof: Parallelogram PQRS is a rectangle.

[Same as in Q4]  $PQ = \frac{1}{2}AC$  ...(1) [Proved as in Q4]

PS joins mid-points of sides AB and AD respectively.

$$PS = \frac{1}{2}BD$$
 ...(2) [Mid-point theorem]  
AC = BD ...(3) [Given]  
From (1), (2) and (3), we get  
PS = PQ  
 $\Rightarrow$  Rectangle PQRS is a square.

#### 6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

**Sol.** ABCD is a parallelogram and diagonal AC bisect  $\angle A$ . We have to show that ABCD is a rhombus.



- 7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.
- **Sol.** Given: A quadrilateral ABCD in which P and Q are the mid-points of the sides AB and CD respectively. AQ intersect DP at S and BQ intersect CP at R.



To prove: PRQS is a parallelogram.

Proof: DC||AB [:: Opposite sides of a parallelogram are parallel]  $\Rightarrow$  AP||OC

$$\Rightarrow \qquad \frac{1}{2}DC = \frac{1}{2}AB$$

DC = -AB

[:: Opposite sides of a parallelogram are equal]

 $\Rightarrow$  QC = AP [:: P is the mid-point of AB and Q is mid-points of CD]

- $\Rightarrow$  APCQ is a parallelogram. [:: AP||QC and QC = AP]
- ∴ AQ||PC [∵ Opposite sides of a ||gm are parallelogram]
- $\Rightarrow$  SQ||PR

Similarly, SP||QR

 $\therefore$  Quadrilateral PRQS is a parallelogram.

Hence, proved.

## 8. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that $\angle A = \angle B$ and $\angle C = \angle D$ .

**Sol.** Given: A quadrilateral ABCD in which AB||CD and AD = BC.



To prove:  $\angle A = \angle B$  and  $\angle C = \angle D$ . Construction: Draw  $DP \perp AB$  and  $CQ \perp AB$ . Proof: In  $\triangle APD$  and  $\triangle BQC$ , we have

 $\angle 1 = \angle 2$  [:: Each equal to 90°]

AB = BC[Given] [Distance between parallel line] So, By RHS criterion of congruence, we have  $\Delta APD \cong \Delta BQC$ [CPCT]  $\angle A = \angle B$ :. Now, DC||AB  $\angle A + \angle 3 = 180^{\circ}$ ...(1) [:: Sum of consecutive interior angles is 180°]  $\angle B + \angle 4 = 180^{\circ}$ ...(2) [Same reason] From (1) and (2), we get  $\angle A + \angle 3 = \angle B + \angle 4$  $[\because \angle A = \angle B]$  $\angle 3 = \angle 4$  $\Rightarrow$  $\angle C = \angle D$  $\Rightarrow$ Hence, proved.

9. In Fig. 8.11, AB || DE, AB = DE, AC||DF and AC = DF. Prove that BC||EF and BC = EF. Sol.



```
Given: AB||DE, AB = DE, AC||DF and AC = DF
To prove: BC||EF and BC = EF
Proof: AC||DF
                             [Given]
       AC = DF
                             [Given]
And
: ACFD is a parallelogram.
\Rightarrow
       AD||CF
                             ...(1) [:: Opposite side of a ||gm are parallel]
       AD||CF
                             ...(2) [:: Opposite sides of a ||gm are equal]
And
Now, AB||DE
                                    [Given]
And
      AB = DE
                                    [Given]
: ABED is a parallelogram.
       AD||BE
                             ...(3) [:: Opposite sides of a are||gm are parallel]
\Rightarrow
And
       AD = BE
                             ...(4) [:: Opposite sides of a ||gm are equal]
From (1) and (3), we get
       CF||BE
And, from (2) and (4) we get
       CF||BE
: BDFE is a parallelogram.
                                    [:: Opposite sides of a ||gm are parallel]
\Rightarrow
       BC||EF
       BC = EF
                                    [:: Opposite sides of a ||gm are equal]
And
```

Hence, proved.

- **10.** E is the mid-point of a median AD of  $\triangle$ ABC and BE is produced to meet AC at F. Show that  $AF = \frac{1}{2}AC$ .
- **Sol.** Given: A  $\triangle$ ABC in which E is the mid-point of median AB and BE is produced to meet AC at F.



To prove:  $AF = \frac{1}{2}AC$ 

Construction: Draw DG||BF intersecting AC at G.

Proof: In  $\triangle$ ADG, E is the mid-point of AD and EF||DG.

...(1) [Converse of mid-point theorem] AF = FG*.*.. In  $\Delta$ FBC, D is the mid-point of BC and DG||BF. ...(2) [Converse of mid-point theorem] FG = GC... From equation (1) and (2), we get AF = FG = GC...(3) AC = AF + FG + GCBut, AC = AF + AF + AF[Using (3)]  $\Rightarrow$ AC = 3AF $\Rightarrow$  $AF = \frac{1}{2}AC$  $\Rightarrow$ 

Hence, proved.

**11.** Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

**Sol.** Given: A square ABCD in which P, Q, R, S are the mid-points of sides AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined. To prove: PQRS is a square. Construction: Join AC and BD. Proof: In  $\triangle ABC$ , P and Q are the mid-points of sides AB and BC respectively.



 $\Rightarrow \angle MPN = \angle MON$   $\Rightarrow \angle MPN = \angle BOA \qquad [\because \angle MPN = \angle BOA]$   $\Rightarrow \angle MPN = 90^{\circ}$ [ $\because$  Diagonals of square are  $\bot \therefore AC \bot BD \Rightarrow \angle BOA = 90^{\circ}$ ]  $\Rightarrow \angle QPS = 90^{\circ}$ Thus, PQRS is a quadrilateral such that PQ = QR = RS = SP and  $\angle QPS = 90^{\circ}$ .

12. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF || AB and  $EF = \frac{1}{2}(AB + CD)$ .

[Hint: Join BE and produce it to meet CD produced at G.]

Sol.



Given: A trapezium ABCD in which E and F are respectively the mid-points of the non-parallel sides AD and BC.

To prove: EF||AB and  $EF = \frac{1}{2}(AB + CD)$ 

Construction: Join DF and produce it to intersect AB produced at G. Proof: In  $\triangle CFD$  and  $\triangle BFG$ , we have

DC||AB  $\therefore \quad \angle C = \angle 3$  [Alternate interior angles] CF = BF  $\angle 1 = \angle 2$  [Vertically opposite angles]

So, By ASA criterion of congruence, we have  $\Delta CFD \cong \Delta BFG$  $\therefore$  CD = BG [CPCT]

∴ CD = BG [CPCT] EF joins mid-points of sides AD and GD respectively

$$\therefore \quad EF||AG \qquad [\because Mid-point theorem] \\ \Rightarrow \quad EF||AB \\ So, \quad EF = \frac{1}{2}AG \qquad [Mid-point theorem] \\ \end{cases}$$

$$\Rightarrow EF = \frac{1}{2}(AB + AG)$$
  
$$\Rightarrow EF = \frac{1}{2}(AB + CD) \qquad [\because CD = BG]$$

Hence, proved.

- 13. Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.
- **Sol.** Given: A quadrilateral ABCD in which bisectors of angles A, B, C, D intersect at P, Q, R, S to from a quadrilateral PQRS.



To Prove: PQRS is a rectangle. Proof: Since ABCD is a parallelogram. Therefore, AB||DC Now, AB||DC and transversal AD intersect them at D and A respectively. Therefore,

	$\angle A + \angle D = 180^{\circ}$	[∵ Sum of consecutive interior angles is 180°]		
$\Rightarrow$	$\frac{1}{2} \angle A = \frac{1}{2} \angle D = 90^{\circ}$			
$\Rightarrow$	$\angle DAS + \angle ADS = 90^{\circ}$	(1)		
		[:: DS and AS are bisectors of $\angle A$ and $\angle D$ respectively]		
But, in	$\Delta DAS$ , we have			
$\angle DAS + \angle ASD + \angle ADS = 180^{\circ}$				
		[ $\because$ Sum of the angles of a triangle is 180°]		
$\Rightarrow \angle 90$	$0^{\circ} + \angle ASD = 180^{\circ}$	[Using (1)]		
$\Rightarrow$	$\angle ASD = 90^{\circ}$			
$\Rightarrow$	$\angle PSR = 90^{\circ}$	[:: $\angle ASD$ and $\angle PSR$ are vertically opposite angles		
		$\therefore \angle PSR = \angle ASD ]$		
Similarly, we can prove that				
	$\angle SRQ = 90^\circ, \angle RQP =$	$=90^\circ$ and $\angle SPQ = 90^\circ$		

Hence, PQRS is a rectangle.

- 14. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.
- **Sol.** ABCD is a parallelogram. Its diagonal AC and BD bisect each other at O. PQ passes through the point of intersection O of its diagonal AC and BD.



In  $\triangle AOP$  and  $\triangle COQ$ , we have

	$\angle 3 = \angle 4$	[Alternate int. $\angle s$ ]
	OA = OC	[Diagonals of a   gm bisects each other]
	$\angle 1 = \angle 2$	[Vertically opposite angles]
<i>.</i>	$\Delta AOP \cong \Delta COQ$	[By ASA Congruence rule]
So,	OP = OQ	[CPCT]
Hence	, PQ is bisected at 0.	

- **15.** ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.
- **Sol.** Given: A rectangle ABCD in which diagonal BD bisects  $\angle B$ .



To prove: ABCD is a square. Proof: DC||AB

[:: Opposite sides of a rectangle are parallel] ..(1) [Alternate interior angles]  $\Rightarrow$  $\angle 4 = \angle 1$ ...(2) [Alternate interior angles] Similarly,  $\angle 3 = \angle 2$ And  $\angle 1 = \angle 2$ ...(3) [Given] From equation (1), (2) and (3), we get  $\angle 3 = \angle 4$ In  $\triangle BDA$  and  $\triangle BDC$ , we have [Given]  $\angle 1 = \angle 2$ BD = BD[Common side]  $\angle 3 = \angle 4$ [proved above] So, By ASA criterion of congruence, we have  $\Delta BDA \cong \Delta BDC$ AB = BC[CPCT] *.*.. So, ABCD is a square. Hence, proved.

- 16. D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangles ABC is divided into four congruent triangles.
- **Sol.** Given: A  $\triangle ABC$  and  $\triangle DEF$  which is formed by joining the mid-point D, E and F of the sides AB, BC and CA of  $\triangle ABC$ .



17. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.





Given: A trapezium ABCD in which E and F are the mid-points of sides AD and BC respectively.

To prove: EF||AB||DC

Construction: Join DF are produce it to intersect AB produced at G. Proof: In  $\triangle DCF$  and  $\triangle GBF$ , we have

$\angle 1 = \angle 2$	[Alternate interior angles because DC  BG]			
$\angle 3 = \angle 4$	[Vertically opposite angles]			
CF = BF	[:: F is the mid-point of BC]			
So, By AAS criterion of congruence, we have				

 $\Delta DCF \cong \Delta GBF$ DF = GF[CPCT] ... In  $\Delta DAG$ , EF joins mid-points of sides DA and DG respectively. *.*•. EF||AG [Mid-point theorem] EF||AB  $\Rightarrow$ AB||DC [Given] But, EF||BC||DC *:*. Hence, proved.

- 18. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.
- **Sol.** ABCD is parallelogram. P is the mid-point of CD. CR which intersects AB at Q is parallel to AP.



In  $\triangle DCR$ , P is the mid-point of CD and AP||CR,

 $\therefore$  A is the mid-points of DR, i.e., AD = AR.

[: The line drawn through the mid-point of one side of a triangle parallel to another side intersects the third side at its mid-point.]

In  $\triangle ARQ$  and  $\triangle BCQ$  we have

	AR = BC	[ $::$ AD = AR (Proved above) and AD = BC]
	$\angle 1 = \angle 2$	[Vertically opposite angles]
	$\angle 3 = \angle 4$	[Alt. $\angle s$ ]
. <b>.</b> .	$\Delta ARQ \cong \Delta BCQ$	[By ASS Congruence rule]
	CQ = QR	[CPCT]
Hence	, $DA = AR$ and $CQ = Q$	R is proved.