

Understanding Quadrilaterals

Exercise 3.1

Question 1:

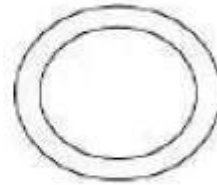
Given here are some figures.



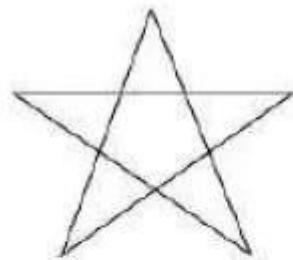
(1)



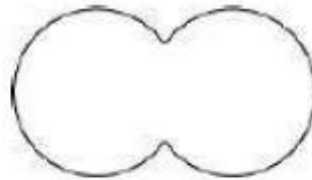
(2)



(3)



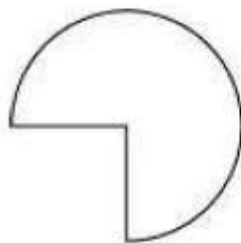
(4)



(5)



(6)



(7)



(8)

Classify each of them on the basis of the following.

- (a) Simple curve
- (b) Simple closed curve
- (c) Polygon
- (d) Convex polygon
- (e) Concave polygon

Answer:

- (a) 1, 2, 5, 6, 7
- (b) 1, 2, 5, 6, 7
- (c) 1, 2
- (d) 2
- (e) 1

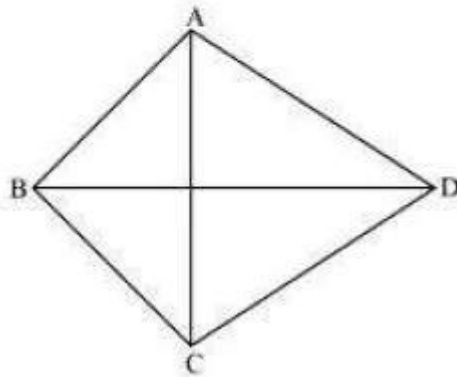
Question 2:

How many diagonals does each of the following have?

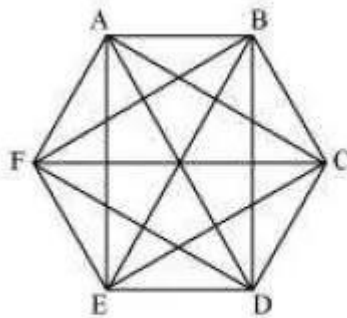
- (a) A convex quadrilateral
- (b) A regular hexagon
- (c) A triangle

Answer:

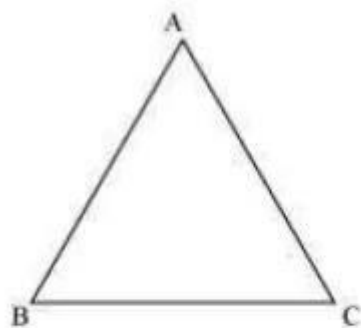
- (a) There are 2 diagonals in a convex quadrilateral.



- (b) There are 9 diagonals in a regular hexagon.



- (c) A triangle does not have any diagonal in it.

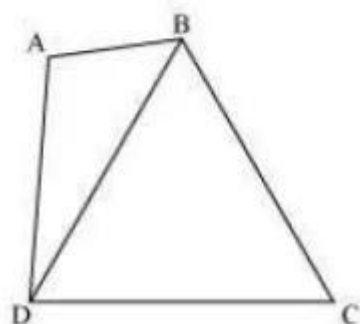


Question 3:

What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

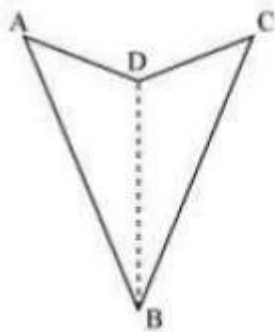
Answer:

The sum of the measures of the angles of a convex quadrilateral is 360° as a convex quadrilateral is made of two triangles.



Here, ABCD is a convex quadrilateral, made of two triangles $\triangle ABD$ and $\triangle BCD$. Therefore, the sum of all the interior angles of this quadrilateral will be same as the sum of all the interior angles of these two triangles i.e., $180^\circ + 180^\circ = 360^\circ$

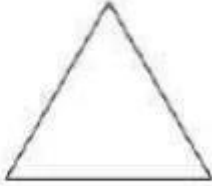
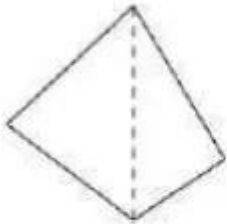
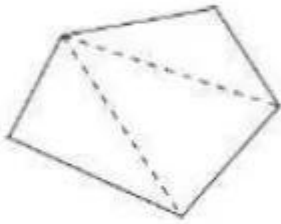
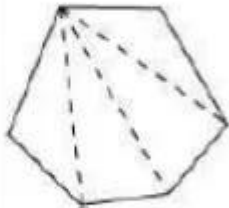
Yes, this property also holds true for a quadrilateral which is not convex. This is because any quadrilateral can be divided into two triangles.



Here again, ABCD is a concave quadrilateral, made of two triangles ΔABD and ΔBCD . Therefore, sum of all the interior angles of this quadrilateral will also be $180^\circ + 180^\circ = 360^\circ$

Question 4:

Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180°	$2 \times 180^\circ$ $= (4 - 2) \times 180^\circ$	$3 \times 180^\circ$ $= (5 - 2) \times 180^\circ$	$4 \times 180^\circ$ $= (6 - 2) \times 180^\circ$

What can you say about the angle sum of a convex polygon with number of sides?

- (a) 7
- (b) 8
- (c) 10
- (d) n

Answer:

From the table, it can be observed that the angle sum of a convex polygon of n sides is $(n - 2) \times 180^\circ$. Hence, the angle sum of the convex polygons having number of sides as above will be as follows.

(a) $(7 - 2) \times 180^\circ = 900^\circ$

(b) $(8 - 2) \times 180^\circ = 1080^\circ$

(c) $(10 - 2) \times 180^\circ = 1440^\circ$

(d) $(n - 2) \times 180^\circ$

Question 5:

What is a regular polygon?

State the name of a regular polygon of

(i) 3 sides

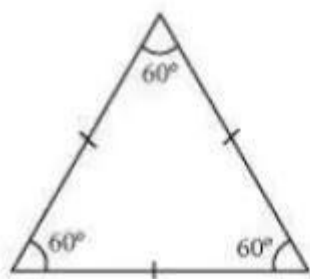
(ii) 4 sides

(iii) 6 sides

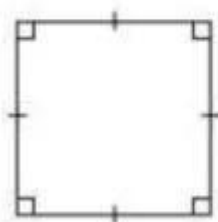
Answer:

A polygon with equal sides and equal angles is called a regular polygon.

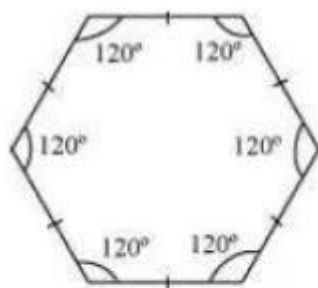
(i) Equilateral Triangle



(ii) Square

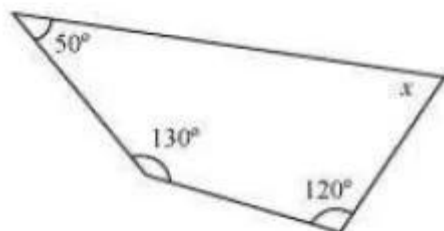


(iii) Regular Hexagon

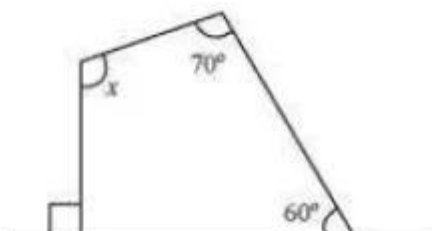


Question 6:

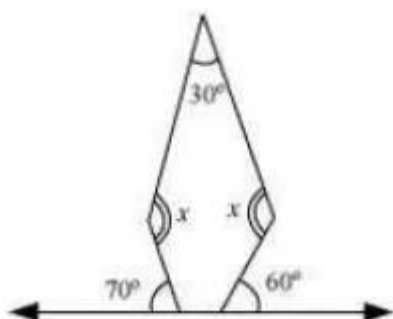
Find the angle measure x in the following figures.



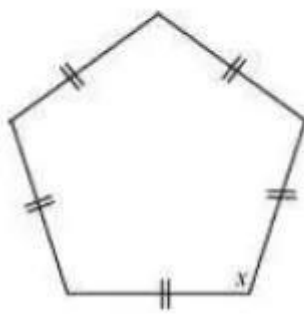
(a)



(b)



(c)



(d)

Answer:

(a)

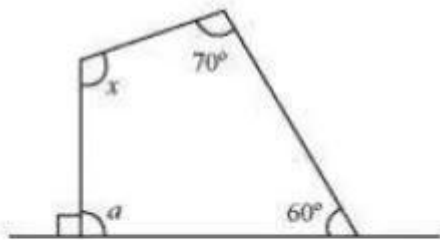
Sum of the measures of all interior angles of a quadrilateral is 360° . Therefore, in the given quadrilateral,

$$50^\circ + 130^\circ + 120^\circ + x = 360^\circ$$

$$300^\circ + x = 360^\circ$$

$$x = 60^\circ$$

(b)



From the figure, it can be concluded that,

$$90^\circ + a = 180^\circ \text{ (Linear pair)}$$

$$a = 180^\circ - 90^\circ = 90^\circ$$

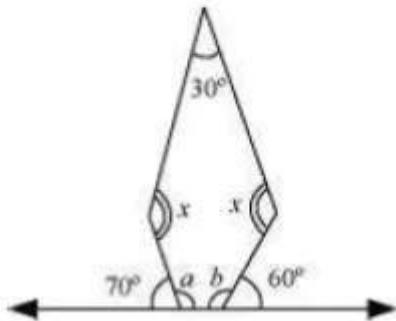
Sum of the measures of all interior angles of a quadrilateral is 360° . Therefore, in the given quadrilateral,

$$60^\circ + 70^\circ + x + 90^\circ = 360^\circ$$

$$220^\circ + x = 360^\circ$$

$$x = 140^\circ$$

(c)



From the figure, it can be concluded that,

$$70 + a = 180^\circ \text{ (Linear pair)}$$

$$a = 110^\circ$$

$$60^\circ + b = 180^\circ \text{ (Linear pair)}$$

$$b = 120^\circ$$

Sum of the measures of all interior angles of a pentagon is 540° .

Therefore, in the given pentagon,

$$120^\circ + 110^\circ + 30^\circ + x + x = 540^\circ$$

$$260^\circ + 2x = 540^\circ$$

$$2x = 280^\circ$$

$$x = 140^\circ$$

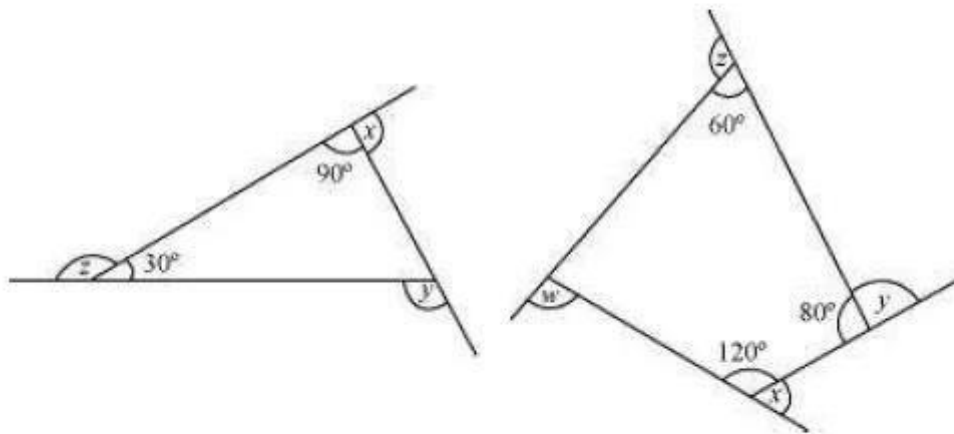
(d)

Sum of the measures of all interior angles of a pentagon is 540° .

$$5x = 540^\circ$$

$$x = 108^\circ$$

Question 7:



(a) find $x + y + z$

(b) find $x + y + z + w$

Answer:

$$(a) x + 90^\circ = 180^\circ \text{ (Linear pair)}$$

$$x = 90^\circ$$

$$z + 30^\circ = 180^\circ \text{ (Linear pair)}$$

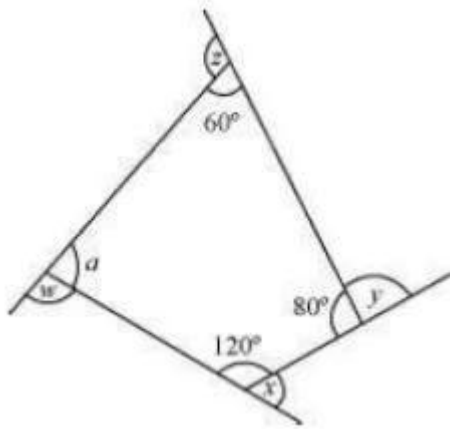
$$z = 150^\circ$$

$$y = 90^\circ + 30^\circ \text{ (Exterior angle theorem)}$$

$$y = 120^\circ$$

$$x + y + z = 90^\circ + 120^\circ + 150^\circ = 360^\circ$$

(b)



Sum of the measures of all interior angles of a quadrilateral is 360° . Therefore, in the given quadrilateral,

$$a + 60^\circ + 80^\circ + 120^\circ = 360^\circ$$

$$a + 260^\circ = 360^\circ$$

$$a = 100^\circ$$

$$x + 120^\circ = 180^\circ \text{ (Linear pair)}$$

$$x = 60^\circ$$

$$y + 80^\circ = 180^\circ \text{ (Linear pair)}$$

$$y = 100^\circ$$

$$z + 60^\circ = 180^\circ \text{ (Linear pair)}$$

$$z = 120^\circ$$

$$w + 100^\circ = 180^\circ \text{ (Linear pair)}$$

$$w = 80^\circ$$

$$\text{Sum of the measures of all interior angles} = x + y + z + w$$

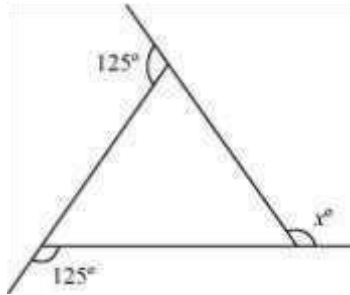
$$= 60^\circ + 100^\circ + 120^\circ + 80^\circ$$

$$= 360^\circ$$

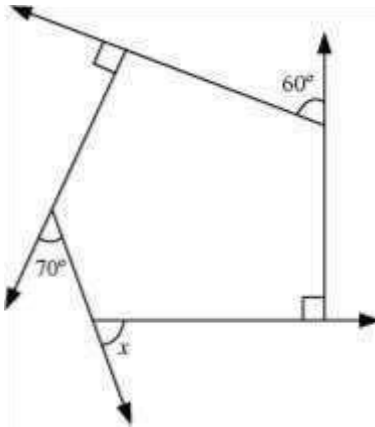
Exercise 3.2 : Solutions of Questions on Page Number : 44

Q1 :

Find x in the following figures.



(a)



(b)

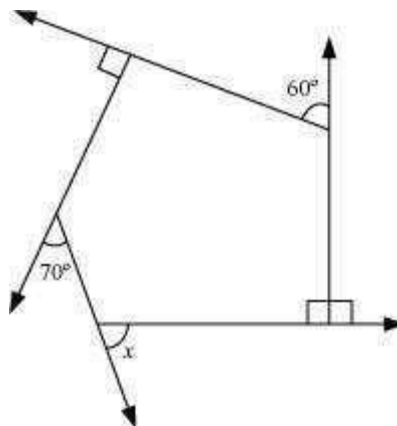
Answer :

We know that the sum of all exterior angles of any polygon is 360° .

(a) $125^\circ + 125^\circ + x = 360^\circ$

$250^\circ + x = 360^\circ$ $x = 110^\circ$

(b)



$60^\circ + 90^\circ + 70^\circ + x + 90^\circ = 360^\circ$

$$310^\circ + x = 360^\circ$$

$$x = 50^\circ$$

Q2 :

Find the measure of each exterior angle of a regular polygon of

(i) 9 sides

(ii) 15 sides

Answer :

(i) Sum of all exterior angles of the given polygon = 360°

Each exterior angle of a regular polygon has the same measure.

Thus, measure of each exterior angle of a regular polygon of 9 sides

$$= \frac{360^\circ}{9} = 40^\circ$$

(ii) Sum of all exterior angles of the given polygon = 360°

Each exterior angle of a regular polygon has the same measure.

Thus, measure of each exterior angle of a regular polygon of 15 sides

$$= \frac{360^\circ}{15} = 24^\circ$$

Q3 :

How many sides does a regular polygon have if the measure of an exterior angle is 24°

Answer :

Sum of all exterior angles of the given polygon = 360°
Measure of each exterior angle = 24°

Thus, number of sides of the regular polygon $= \frac{360^\circ}{24^\circ} = 15$

Q4 :

How many sides does a regular polygon have if each of its interior angles is 165°

Answer :

Measure of each interior angle = 165°

Measure of each exterior angle = $180^\circ - 165^\circ = 15^\circ$

The sum of all exterior angles of any polygon is 360° .

Thus, number of sides of the polygon $= \frac{360^\circ}{15^\circ} = 24$

Q5 :

**(a) Is it possible to have a regular polygon with measure of each exterior angle as 22° (b)
Can it be an interior angle of a regular polygon Why**

Answer :

The sum of all exterior angles of all polygons is 360° . Also, in a regular polygon, each exterior angle is of the same measure. Hence, if 360° is a perfect multiple of the given exterior angle, then the given polygon will be possible.

(a) Exterior angle = 22°

360° is not a perfect multiple of 22° . Hence, such polygon is not possible.

(b) Interior angle = 22°

$$\text{Exterior angle} = 180^\circ - 22^\circ = 158^\circ$$

Such a polygon is not possible as 360° is not a perfect multiple of 158° .

Q6 :

(a) What is the minimum interior angle possible for a regular polygon

(b) What is the maximum exterior angle possible for a regular polygon

Answer :

Consider a regular polygon having the lowest possible number of sides (i.e., an equilateral triangle). The exterior angle of this triangle will be the maximum exterior angle possible for any regular polygon.

Exterior angle of an equilateral triangle

$$= \frac{360^\circ}{3} = 120^\circ$$

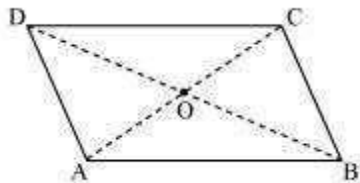
Hence, maximum possible measure of exterior angle for any polygon is 120° . Also, we know that an exterior angle and an interior angle are always in a linear pair.

$$\text{Hence, minimum interior angle} = 180^\circ - 120^\circ = 60^\circ$$

Exercise 3.3 : Solutions of Questions on Page Number : 50

Q1 :

Given a parallelogram ABCD. Complete each statement along with the definition or property used.



(i) $AD = \dots$

(ii) $\angle DCB = \dots$

(iii) $OC = \dots$

(iv) $m\angle DAB + m\angle CDA = \dots$

Answer :

(i) In a parallelogram, opposite sides are equal in length.

$$AD = BC$$

(ii) In a parallelogram, opposite angles are equal in measure.

$$\angle DCB = \angle DAB$$

(iii) In a parallelogram, diagonals bisect each other.

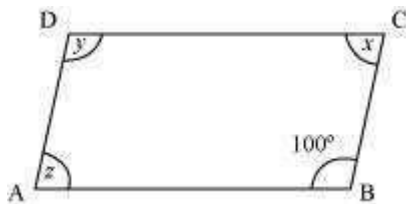
$$\text{Hence, } OC = OA$$

(iv) In a parallelogram, adjacent angles are supplementary to each other.

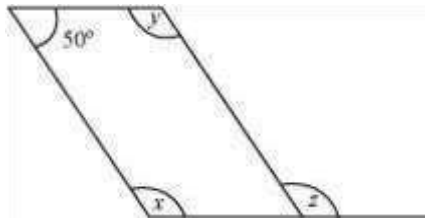
$$\text{Hence, } m\angle DAB + m\angle CDA = 180^\circ$$

Q2 :

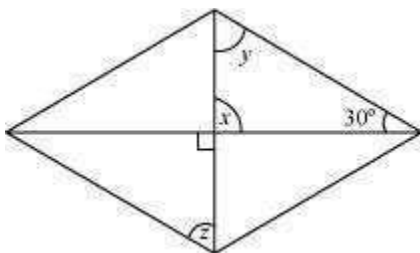
Consider the following parallelograms. Find the values of the unknowns x, y, z .



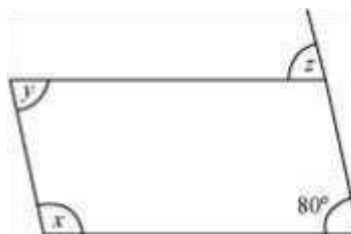
(i)



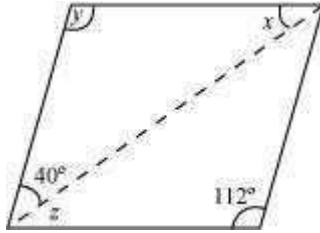
(ii)



(iii)



(iv)



(v)

Answer :

(i) $x + 100^\circ = 180^\circ$ (Adjacent angles are supplementary) $x = 80^\circ$ $z = x = 80^\circ$ (Opposite angles are equal) $y = 100^\circ$ (Opposite angles are equal)

(ii) $50^\circ + y = 180^\circ$ (Adjacent angles are supplementary) $y = 130^\circ$ $x = y = 130^\circ$ (Opposite angles are equal) $z = x = 130^\circ$ (Corresponding angles) (iii) $x = 90^\circ$ (Vertically opposite angles) $x + y + 30^\circ = 180^\circ$ (Angle sum property of triangles)

$120^\circ + y = 180^\circ$ $y = 60^\circ$ $z = y = 60^\circ$ (Alternate interior angles) (iv) $z = 80^\circ$ (Corresponding angles) $y = 80^\circ$ (Opposite angles are equal) $x + y = 180^\circ$ (Adjacent angles are supplementary) $x = 180^\circ - 80^\circ = 100^\circ$

(v) $y = 112^\circ$ (Opposite angles are equal) $x + y + 40^\circ = 180^\circ$ (Angle sum property of triangles) $x + 112^\circ +$

$$40^\circ = 180^\circ - x + 152^\circ = 180^\circ - x = 28^\circ \quad z = x = 28^\circ$$

(Alternate interior angles)

Q3 :

Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180^\circ$

(ii) $AB = DC = 8 \text{ cm}$, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$

(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$

Answer :

(i) For $\angle D + \angle B = 180^\circ$, quadrilateral ABCD may or may not be a parallelogram. Along with this condition, the following conditions should also be fulfilled.

The sum of the measures of adjacent angles should be 180° .

Opposite angles should also be of same measures.

(ii) No. Opposite sides AD and BC are of different lengths.

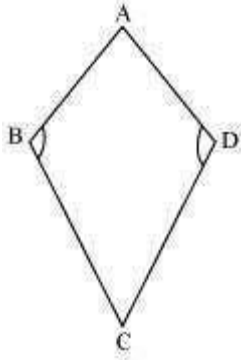
(iii) No. Opposite angles A and C have different measures.

Q4 :

Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Answer :

Here, quadrilateral ABCD (kite) has two of its interior angles, $\angle B$ and $\angle D$, of same measures. However, still the quadrilateral ABCD is not a parallelogram as the measures of the remaining pair of opposite angles, $\angle A$ and $\angle C$, are not equal.



Q5 :

The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Answer :

Let the measures of two adjacent angles, $\angle A$ and $\angle B$, of parallelogram ABCD are in the ratio of 3:2. Let $\angle A = 3x$ and $\angle B = 2x$

We know that the sum of the measures of adjacent angles is 180° for a parallelogram.

$$\angle A + \angle B = 180^\circ$$

$$3x + 2x = 180^\circ$$

$$5x = 180^\circ$$

$$x = \frac{180^\circ}{5} = 36^\circ$$

$$\angle A = \angle C = 3x = 108^\circ \text{ (Opposite angles)}$$

$$\angle B = \angle D = 2x = 72^\circ \text{ (Opposite angles)}$$

Thus, the measures of the angles of the parallelogram are 108° , 72° , 108° , and 72° .

Q6 :

Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Answer :

Sum of adjacent angles = 180°

$$\angle A + \angle B = 180^\circ$$

$$2\angle A = 180^\circ (\angle A = \angle B)$$

$$\angle A = 90^\circ$$

$$\angle B = \angle A = 90^\circ$$

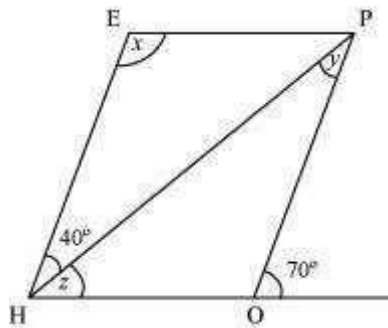
$$\angle C = \angle A = 90^\circ (\text{Opposite angles})$$

$$\angle D = \angle B = 90^\circ (\text{Opposite angles})$$

Thus, each angle of the parallelogram measures 90° .

Q7 :

The adjacent figure HOPE is a parallelogram. Find the angle measures x , y and z . State the properties you use to find them.



Answer :

$$y = 40^\circ (\text{Alternate interior angles})$$

$$70^\circ = z + 40^\circ (\text{Corresponding angles})$$

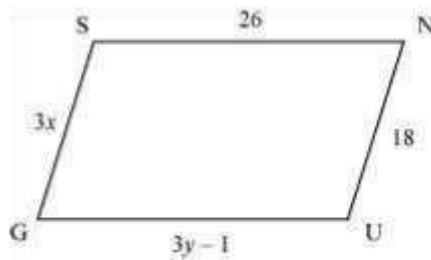
$$70^\circ - 40^\circ = z \quad z = 30^\circ \quad x + (z + 40^\circ) = 180^\circ$$

$$(\text{Adjacent pair of angles}) \quad x + 70^\circ = 180^\circ \quad x =$$

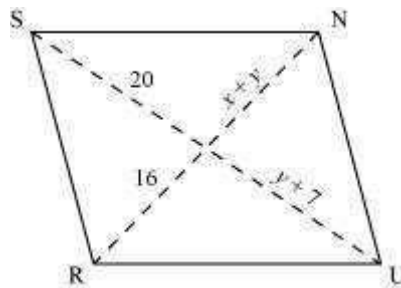
$$110^\circ$$

Q8 :

The following figures GUNS and RUNS are parallelograms. Find x and y . (Lengths are in cm)



(i)



(ii)

Answer :

(i) We know that the lengths of opposite sides of a parallelogram are equal to each other.

$$GU = SN$$

$$3y - 1 = 26$$

$$3y = 27$$

$$y = 9$$

$$SG = NU$$

$$3x = 18 \quad x =$$

$$= 6$$

Hence, the measures of x and y are 6 cm and 9 cm respectively. (ii) We know that the diagonals of a parallelogram bisect each other.

$$y + 7 = 20 \quad y$$

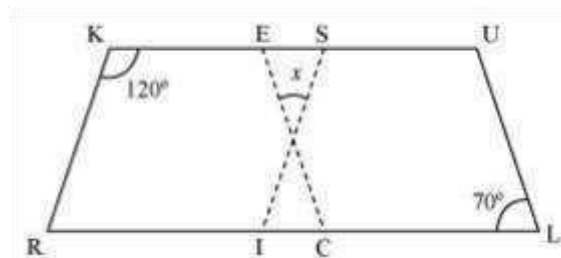
$$= 13 \quad x + y =$$

$$16 \quad x + 13 =$$

$$16 \quad x = 3$$

Hence, the measures of x and y are 3 cm and 13 cm respectively.

Q9 :



In the above figure both RISK and CLUE are parallelograms. Find the value of x .

Answer :

Adjacent angles of a parallelogram are supplementary.

In parallelogram RISK, $\angle RKS + \angle ISK = 180^\circ$

$$120^\circ + \angle ISK = 180^\circ$$

$$\angle ISK = 60^\circ$$

Also, opposite angles of a parallelogram are equal.

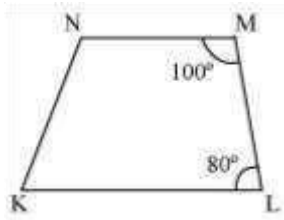
In parallelogram CLUE, $\angle ULC = \angle CEU = 70^\circ$

The sum of the measures of all the interior angles of a triangle is 180° .

$$x + 60^\circ + 70^\circ = 180^\circ \quad x = 50^\circ$$

Q10 :

Explain how this figure is a trapezium. Which of its two sides are parallel



Answer :

If a transversal line is intersecting two given lines such that the sum of the measures of the angles on the same side of transversal is 180° , then the given two lines will be parallel to each other.

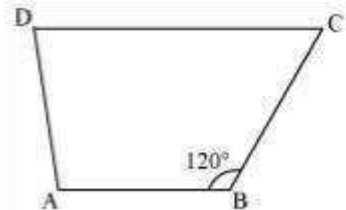
$$\text{Here, } \angle NML + \angle MLK = 180^\circ$$

Hence, $NM \parallel LK$

As quadrilateral KLMN has a pair of parallel lines, therefore, it is a trapezium.

Q11 :

Find $m\angle C$ in the following figure if $\overline{AB} \parallel \overline{DC}$



Answer :

Given that, $\overline{AB} \parallel \overline{DC}$

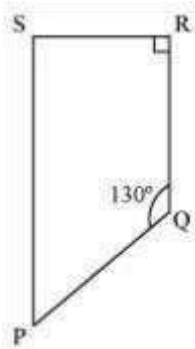
$\angle B + \angle C = 180^\circ$ (Angles on the same side of transversal)

$$120^\circ + \angle C = 180^\circ$$

$$\angle C = 60^\circ$$

Q12 :

Find the measure of $\angle P$ and $\angle S$, if $\overline{SP} \parallel \overline{RQ}$ in the following figure. (If you find $m\angle R$, is there more than one method to find $m\angle P$)



Answer :

$\angle P + \angle Q = 180^\circ$ (Angles on the same side of transversal)

$$\angle P + 130^\circ = 180^\circ$$

$$\angle P = 50^\circ$$

$\angle R + \angle S = 180^\circ$ (Angles on the same side of transversal)

$$90^\circ + \angle R = 180^\circ$$

$$\angle S = 90^\circ$$

Yes. There is one more method to find the measure of $m\angle P$.

$m\angle R$ and $m\angle Q$ are given. After finding $m\angle S$, the angle sum property of a quadrilateral can be applied to find $m\angle P$.

Exercise 3.4 : Solutions of Questions on Page Number : 55

Q1 :

State whether True or False.

- (a) All rectangles are squares.**
- (b) All rhombuses are parallelograms.**
- (c) All squares are rhombuses and also rectangles.**
- (d) All squares are not parallelograms.**
- (e) All kites are rhombuses.**
- (f) All rhombuses are kites.**
- (g) All parallelograms are trapeziums.**
- (h) All squares are trapeziums.**

Answer :

- (a) False. All squares are rectangles but all rectangles are not squares.
- (b) True. Opposite sides of a rhombus are equal and parallel to each other.
- (c) True. All squares are rhombuses as all sides of a square are of equal lengths. All squares are also rectangles as each internal angle measures 90° .
- (d) False. All squares are parallelograms as opposite sides are equal and parallel.
- (e) False. A kite does not have all sides of the same length.
- (f) True. A rhombus also has two distinct consecutive pairs of sides of equal length.
- (g) True. All parallelograms have a pair of parallel sides.
- (h) True. All squares have a pair of parallel sides.

Q2 :

Identify all the quadrilaterals that have

(a) four sides of equal length

(b) four right angles

Answer :

(a) Rhombus and Square are the quadrilaterals that have 4 sides of equal length.

(b) Square and rectangle are the quadrilaterals that have 4 right angles.

Q3 :

Explain how a square is.

(i) a quadrilateral

(ii) a parallelogram

(iii) a rhombus

(iv) a rectangle

Answer :

(i) A square is a quadrilateral since it has four sides.

(ii) A square is a parallelogram since its opposite sides are parallel to each other. (iii) A square is a rhombus since its four sides are of the same length.

(iv) A square is a rectangle since each interior angle measures 90° .

Q4 :

Name the quadrilaterals whose diagonals.

(i) bisect each other

(ii) are perpendicular bisectors of each other

(iii) are equal

Answer :

(i) The diagonals of a parallelogram, rhombus, square, and rectangle bisect each other.

(ii) The diagonals of a rhombus and square act as perpendicular bisectors.

(iii) The diagonals of a rectangle and square are equal.

Q5 :

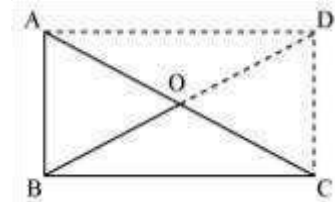
Explain why a rectangle is a convex quadrilateral.

Answer :

In a rectangle, there are two diagonals, both lying in the interior of the rectangle. Hence, it is a convex quadrilateral.

Q6 :

ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Answer :

Draw lines AD and DC such that $AD \parallel BC$, $AB \parallel DC$

$AD = BC$, $AB = DC$

ABCD is a rectangle as opposite sides are equal and parallel to each other and all the interior angles are of 90° .

In a rectangle, diagonals are of equal length and also these bisect each other.

Hence, $AO = OC = BO = OD$

Thus, O is equidistant from A, B, and C.