Polynomials

Exercise 2.1

Write the correct answer in each of the following:

1. Which of the following is a polynomials?

(a) $\frac{x^2}{2}-\frac{2}{x^2}$

(b) $\sqrt{2x}-1$

(c) $x^2+\frac{3x^{\frac{2}{3}}}{\sqrt{x}}$

(d) $\frac{x-1}{x+1}$

Sol. (a) $\frac{x^2}{2}-\frac{2}{x^2}=\frac{x^2}{2}-2x^{-2}$

Second term is $-2x^{-2}$. Exponent of x^{-2} is – 2, which is not a whole number.
So, this algebraic expression is not a polynomial.

(b) $\sqrt{2x}-1=\sqrt{2}x^{\frac{1}{2}}-1$

First term is $\sqrt{2}x^{\frac{1}{2}}$. Here, the exponent of the second term, i.e., $x^{\frac{1}{2}}$ is $\frac{1}{2}$, which is not a whole number. So, this algebraic expression is not a polynomial.

(c) $x^2+\frac{3x^{\frac{2}{3}}}{\sqrt{x}}=x^2+3x$

In this expression, we have only whole number as the exponent of the variable in each them. Hence, the given algebraic expression is a polynomial.

2. $\sqrt{2}$ is a polynomial of degree

(a) 2

(b) 0

(c) 1

(d) $\frac{1}{2}$

Sol. $\sqrt{2}$ is a constant polynomial. The only term here is $\sqrt{2}$ which can be written as $\sqrt{2}x^0$.
So, the exponent of x is zero. Therefore, the degree of the polynomial is 0.
Hence, (b) is the correct answer.

3. Degree of the polynomial of $4x^4+0x^3+0x^5+5x+7$ is

(a) 4

(b) 5

(c) 3

(d) 7

Sol. The height power of the variable in a polynomial is called the degree of the polynomial. In this polynomial, the term with highest power of x is $4x^4$. Highest power of x is 4, so the degree of the given polynomial is 4.

4. Degree of the zero polynomial

(a) 0
(b) 1
(c) Any natural number
(d) Not defined.

Sol. Degree of the zero degree polynomial (0) is not defined.
Hence, (d) is the correct answer.

5. If $p(x) = x^2 - 2\sqrt{2x} + 1,$ then $p(2\sqrt{2})$ is equal to

(a) 0
(b) 1
(c) $4\sqrt{2}$
(d) $8\sqrt{2} + 1$

Sol. We have $p(x) = x^2 - 2\sqrt{2x} - 1$

$\therefore \quad p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2} + 1)$

$= 8 - 8 + 1$

= 1

Hence, (b) is the correct answer.

6. The value of the polynomial $5x - 4x^2 + 3,$ when $x = -1$ is

(a) – 6
(b) 6
(c) 2
(d) – 2

Sol. Let $P(x) = 5x - 4x^2 + 3$

Therefore, $P(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

Hence, (a) is the correct answer.

7. If $p(x) = x + 3,$ then $p(x) + p(-x)$ is equal to

(a) 3
(b) 2x
(c) 0
(d) 6

Sol. We have $p(x) = x + 3,$ then

$$p(-x) = -x + 3$$

Therefore, $p(x) + P(-x) = x + 3 + (-x + 3) = x + 3 - x + 3 = 6$

Hence, (d) is the correct answer.

8. Zero of the zero polynomial is

(a) 0
(b) 1
(c) Any real number
(d) Not defined

Sol. The zero (or degree) of the zero polynomial is undefined.
Hence, (d) is the correct answer.

9. Zero of the polynomial $p(x)=2x+5$ is

(a) $-\frac{2}{5}$
(b) $-\frac{5}{2}$
(c) $\frac{2}{5}$
(d) $\frac{5}{2}$

Sol. Finding a zero of p(x) is the same as solving an equation P(x) = 0.
Now, $p(x)=0 \Rightarrow 2x+5=0,$
Which give us $x=-\frac{5}{2}.$
Therefore, $-\frac{5}{2}$ is the zero of the polynomial.
Hence, (b) is the correct answer.

10. One of the zeroes of the polynomial $2x^2+7x-4$ is

(a) 2
(b) $\frac{1}{2}$
(c) $-\frac{1}{2}$
(d) -2

Sol. We have $p(x)=2x^2+7x+4$

(a) $p(2)=2(2)^2+7(2)-4$
$=8+14-4$
$=18 \neq 0$

(b) $p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^2+7\left(\frac{1}{2}\right)-4$
$=2\times\frac{1}{4}+\frac{7}{2}-4=\frac{1}{2}+\frac{7}{2}-4=4-4=0$

(c) $p\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^2+7\left(-\frac{1}{2}\right)-4$

$$=2\times\frac{1}{4}-\frac{7}{2}-4=\frac{1}{2}-\frac{7}{2}-4$$
$$=-3-4$$
$$=-7\neq 0$$

(d) $p(-2)=2(-2)^2+7(-2)-4$

$$=8-14-4=-10\neq 0$$

As $p\left(\frac{1}{2}\right)=0,$ we say that $\frac{1}{2}$ is a zero of the polynomial. Hence, $\frac{1}{2}$ is one of the zero of the polynomial $2x^2+7x-4.$

Hence, (b) is the correct answer.

11. If $x^{51}+51$ is divided by x + 1, the remainder is

(a) 0
(b) 1
(c) 49
(d) 50

Sol. If p(x) is divided by x + a, then the remainder is p(-a).

Here $p(x)=x^{51}+51$ is divided by x +1, then

Remainder $=p(-1)=(-1)^{51}+51=50=-1+51=50$

Hence, (d) is the correct answer.

12. If x +1, is a factor of the polynomial $2x^2+kx,$ then the value of k is

(a) – 3
(b) 4
(c) 2
(d) – 2

Sol. Let $p(x)=2x^2=kx$

If x + 1 is a factor of p(x), then by factor theorem $p(-1)=0$

Now, $p(-1)=0\Rightarrow 2(-1)^2+k(-1)=0$

$\Rightarrow$ $2-k=0;k=2$

Hence, (c) is the correct answer.

13. x + 1, is a factor of the polynomial

(a) x^3+x^2-x+1
(b) x^3+x^2+x+1
(c) $x^4+x^3+x^2+1$
(d) $x^4+3x^3+3x^2+x+1$

Sol. If x +1 is a factor of p(x), then p (-1) = 0

(a) Let $p(x) = x^3 + x^2 - x + 1$

$\therefore \quad p(-1) = (-1)^3 + (-1)^2 - (-1) + 1$

$= -1 + 1 + 1 + 1 = 2 \neq 0$

So, x + 1 is not a factor of p(x).

(b) Let $p(x) = x^3 + x^2 + x + 1$

$\therefore \quad p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

(c) Let $p(x) = x^4 + x^3 + x^2 + 1$

$\therefore \quad p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 + 1 = 2 \neq 0$

(d) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$\therefore \quad p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0$

Hence, x +1 is a factor of $x^3 + x^2 + x + 1$.

So, (b) is the correct answer.

14. One of the factor of $(25x^2 - 1) + (1 + 5x)^2$ is

(a) 5 + x

(b) 5 – x

(c) 5x – 1

(d) 10x

Sol. $(25x^2 - 1) + (1 + 5x)^2 = (5x)^2 - 1^2 + (5x + 1)^2$

$= (5x - 1)(5x - 1) + (5x + 1)^2 = (5x + 1)(5x - 1 + 5x + 1)$

$= (5x + 1)(10x) = 10x(5x + 1)$

Hence, one of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is 10x. Therefore, (d) is the correct answer.

15. The value of $249^2 - 248^2$ is

(a) 1^2

(b) 477

(c) 487

(d) 497

Sol. $(249)^2 - (248)^2 = (249 + 248)(249 - 248)$

= (497) (1) = 497

Hence, (d) is the correct answer.

16. The factorization of $= 4x^2 + 8x + 3$ is

(a) $(x + 1)(x + 3)$

(b) $(2x + 1)(2x + 3)$

(c) $(2x+2)(2x+5)$

(d) $(2x-1)(2x-3)$

Sol. $4x^2+8x+3=4x^2+6x+2x+3$

$=2x(2x+3)+1(2x+3)=(2x+1)(2x+3)$

Hence, (b) is the correct answer.

17. Which of the following is a factor of $(x+y)^2-(x^3+y^3)$?

(a) x^2+y^2+2xy

(b) x^2+y^2-xy

(c) xy^2

(d) $3xy$

Sol. $(x+y)^3-(x^3+y^3)=x^3+y^3+3xy(x+y)-x^3-y^3$

$=3xy(x+y)$

So, 3xy is a factor of $(x+y)^3-(x^3+y^3)$.

Hence, (d) is the correct answer.

18. The coefficient of x in the expansion of $(x+3)^3$ is

(a) 1

(b) 9

(c) 18

(d) 27

Sol. Using $(a+b)^3=a^3+b^3+3ab(a+b)$, we get

$(x+3)^3=x^3+3^3+3\times x\times 3(x+3)$

$=x^3+27+9x^2\times 27x$

Therefore, the coefficient of x is 27.

Hence, (d) is the correct answer.

19. If $\frac{x}{y}+\frac{y}{x}=-1$ the value of x^3-y^3 is

(a) 1

(b) – 1

(c) 0

(d) $\frac{1}{2}$

Sol. $\frac{x}{y}+\frac{y}{x}=-1\Rightarrow\frac{x^2+y^2}{xy}=-1$

$\Rightarrow\quad x^2+y^2=-xy$

Now, $x^3-y^3=(x-y)(x^2+y^2+xy)$

$=(x-y)(-xy+xy)\quad[\because x^2+y^2=-xy]$

$= (x - y)(0)$

= 0

Hence, (c) is the correct answer.

20. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is

(a) 0

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

Sol. $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$

$\Rightarrow \quad 49x^2 - b = (7x)^2 - \left(\frac{1}{2}\right)^2$

$= 49^2 - \frac{1}{4} \qquad [\because (a+b)(a-b) = a^2 - b^2]$

So, we get $b = \frac{1}{4}$.

Hence, (c) is the correct answer.

21. If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is equal to

(a) 0

(b) abc

(c) 3abc

(d) 2abc

Sol. We know that

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

As $a + b + c = 0$, so, $a^3 + b^3 + c^3 - 3abc = (0)\ (a^2 + b^2 + c^2 - ab - bc - ca) = 0$

Hence, $a^3 + b^3 + c^3 = 3abc$.

Therefore, (c) 3abc is the correct answer.

Polynomials
<u>Exercise 2.2</u>

1. Which of the following expression are polynomials? Justify your answer.

(i) 8

(ii) $\sqrt{3}x^2 - 2x$

(iii) $1-\sqrt{5}x$

(iv) $\frac{1}{5x^{-2}}+5x+7$

(v) $\frac{(x-2)(x-4)}{x}$

(vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$

(viii) $\frac{1}{2x}$

Sol. (i) 8 is a constant polynomial.

(ii) $\sqrt{3}x^2 - 2x$

In each term of this expression, the exponent of the variable x is a whole number. Hence, it is a polynomial.

(iii) $1-\sqrt{5}x = 1-\sqrt{5}x^{\frac{1}{2}}$

Here, the exponent of the second term, i.e., $x^{\frac{1}{2}}$, $\frac{1}{2}$, which is not a whole number Hence, the given algebraic expression is not a polynomial.

(iv) $\frac{1}{5x^{-2}}+5x+7=\frac{1}{5}x^2+5x+7$

In each term of this expression, the exponent of the variable x is a whole number. Hence, it is a polynomial.

(v) $\frac{(x-2)(x-4)}{x}=\frac{x^2-6x+8}{x}=x-6+\frac{8}{x}=x-6+8x^{-1}$

Here, the exponent of variable x in the third term, i.e., in $8x^{-1}$, is – 1, which is not a whole number. So, this algebraic expression is not a polynomial.

(vi) $\frac{1}{x+1}=(x+1)^{-1}$ which cannot be reduced to an expression in which the exponent of the variable x have only whole numbers in each of its terms. So, this algebraic expression is not a polynomial.

(vii) $\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$

In this expression, the exponent of a in each term is a whole number, so this expression is a polynomial.

(viii) $\frac{1}{2x} = \frac{1}{2}x^{-1}$

Here, the exponent of the variable x is – 1, which is not a whole number so, this algebraic expression is not a polynomial.

2. Write whether the following statements are True or False. Justify your answer.

(i) A binomial can have at most two terms

(ii) Every polynomial is a binomial

(iii) A binomial may have degree 5

(iv) Zero of a polynomial is always 0

(v) A polynomial cannot have more than one zero

(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

Sol. (i) The given statement is false because binomial have exactly two terms.

(ii) A polynomial can be a monomial, binomial trinomial or can have finite number of terms. For example, $x^4 + x^3 + x^2 + 1$ is a polynomial but not binomial.

Hence, the given statement is false.

(iii) The given statement is true because a binomial is a polynomial whose degree is a whole number ≥ 1. For example, $x^5 - 1$ is a binomial of degree 5.

(iv) The given statement is false, because zero of polynomial can be any real number.

(v) The given statement is false, because a polynomial can have any number of zeroes which depends on the degree of the polynomial.

(vi) The given statement is false. For example, consider the two polynomial $-x^5 + 3x^2 + 4$ and $x^5 + x^4 + 2x^3 + 3.$ The degree of each of these polynomial is 5. Their sum is $x^4 + 2x^3 + 3x^2 + 7.$ The degree of this polynomial is not 5.

Polynomials
Exercise 2.3

1. **Classify the following polynomial as polynomials in one variable, two variable etc.**
(i) x^2+x+1
(ii) y^3-5y
(iii) $xy+yz+zx$
(iv) $x^2-2xy+y^2+1$

Sol. (i) x^2+x+1 is a polynomial in one variable.
(ii) y^3-5y is a polynomial in one variable.
(iii) $xy+yz+zx$ is a polynomial in three variable.
(iv) $x^2-2xy+y^2+1$ is a polynomial in three variable.

2. **Determine the degree of each of the following polynomials:**
(i) 2x – 1
(ii) – 10
(iii) $x^3-9x+3x^5$
(iv) $y^3(1-y^4)$

Sol. (i) Since the highest power of x is 1, the degree of the polynomial 2x – 1 is 1.
(ii) – 10 is a non-zero constant. A non-zero constant term is always regarded as having degree 0.
(iii) Since the highest power of x is 5, the degree of the polynomial $x^3-9x+3x^5$ is 5.
(iv) $y^3(1-y^4)=y^3-y^7$ Since the highest power of y is 7, the degree of the polynomial is 7.

3. **For the polynomial** $\frac{x^2+2x+1}{5}-\frac{7}{2}x^2-x^6$, **write**
(i) the degree of the polynomial
(ii) the coefficient of x^3.
(iii) the coefficient of x^6.
(iv) the constant term.

Sol. (i) We know that highest power of variable in a polynomial is the degree of the polynomial.
In the given polynomial, the term with highest of x is $-x^6$, and the exponent of x in this term in 6.
(ii) The coefficient of x^3 is $\frac{1}{5}$.
(iii) The coefficient of x^6 is – 1.
(iv) The constant term is $\frac{1}{5}$.

4. Write the coefficient of x^2 in each of the following:

(i) $\frac{\pi}{6}x+x^2-1$

(ii) $3x-5$

(iii) $(x-1)(3x-4)$

(iv) $(2x-5)(2x^2-3x+1)$

Sol. (i) The coefficient of x^2 in the given polynomial is 1.

(ii) The given polynomial can be written as 0. $x^2+3x-5.$ So, the coefficient of x^2 in the given polynomial is 0.

(iii) The given polynomial can be written as:

$$(x-1)(3x-4)=3x^2-4x-3x+4$$
$$=3x^2-7x+4$$

So, coefficient of x^2 in the given polynomial is 3.

(iv) The given polynomial can be written as:

$$(2x-5)(2x^2-3x+1)=4x^3-6x^2+2x-10x^2+15x-5$$
$$=4x^3-16x^2+17x-5$$

So, the coefficient of x^2 in the given polynomial is – 16.

5. Classify the following as a constant, linear quadratic and cubic polynomials:

(i) $2-x^2+x^3$

(ii) $3x^3$

(iii) $5t-\sqrt{7}$

(iv) $4-5y^2$

(v) 3

(vi) $2+x$

(vii) y^3-y

(viii) $1+x+x^3$

(ix) t^2

(x) $\sqrt{2}x-1$

Sol. We know that

(a) a polynomial in which exponent of the variable is zero, is called a constant term.

Here, (v) 3 is a constant polynomial because $3=3x^0$, exponent of the variable x is 0.

(b) a polynomial of degree 1 is called a linear polynomial.

$5t-\sqrt{7}, 2+x$ and $\sqrt{2}x+1$ are linear polynomial.

(c) A polynomial of degree 2 is called a quadratic polynomial.

$4-5y^2, 1+x+x^2$ and t^2 are quadratic polynomials.

(d) A polynomial of degree 3 is called a cubic polynomial.

$2-x+x^3, 3x^3$ and y^3-y are cubic polynomials.

6. **Give an example of a polynomial, which is:**

(i) monomial of degree 1.
(ii) binomial of degree 20.
(iii) trinomial of degree 2.

Sol. We know that a polynomial having only one term is called a monomial, a polynomial having only two terms is called binomial, a polynomial having only three terms is called a trinomial.

(i) 3x is monomial of degree 1.
(ii) $x^{20}-7$ is a binomial of degree 20.
(iii) $5x^2+3x-1$ is a trinomial of degree 2.

7. **Find the value of the polynomial** $3x^3-4x^2+7x+5,$ **when x = 3 and also when x = - 3.**

Sol. Let $p(x)=3x^2-4x^2+7x-5$

$\therefore \quad p(3)=3(3)^3-4(3)^2+7(3)-5$
= 3(27) – 4(9) + 21 – 5
= 81 – 36 + 21 – 5
= 61

Now, $p(-3)=3(-3)^3-4(-3)^2+7(-3)-5$
$=3(-27)-4(9)-21-5$
= – 81 – 36 – 21 – 5
= – 143

8. **If** $p(x)=x^2-4x+3,$ **evaluate** $p(2)-p(-1)+p\left(\frac{1}{2}\right)$

Sol. We have $p(x)=x^2-4x+3$

$\therefore p(2)-p(-1)+p\left(\frac{1}{2}\right)=(2^2-4\times2+3)-\{(-1)^2-4(-1)+3\}+\left\{\left(\frac{1}{2}\right)^2-4\times\frac{1}{2}+3\right\}$

$=(4-8+3)-(1+4+3)+\left(\frac{1}{4}-2+3\right)$

$=-1-8+\frac{5}{4}$

$=-9+\frac{5}{4}=\frac{-36+5}{4}=\frac{-31}{4}$

9. **Find p (0), p(1), p(- 2) for the following polynomials:**

(i) $p(x)=10x-4x^2-3$
(ii) $p(y)=(y+2)(y-2)$

Sol. (i) We have $p(x)=10x-4x^2-3$

$\therefore \quad p(0)=10(0)-4(0)^2-3$

$= 0 - 0 - 3 = -3$

And, $p(1) = 10(1) - 4(1)^2 - 3$

$= 10 - 4 - 3 = 10 - 7 = 3$

And, $P(-2) = 10(-2) - 4(-2)^2 - 3$

$= -20 - 4(4) - 3 = -20 - 16 - 3 = -39$

(ii) We have $p(y) = (y+2)(y-2) = y^2 - 4$

$\therefore$ $p(0) = (0)^2 - 4$

$= 0 - 4 = -4$

And, $p(1) = (1)^2 - 4$

$= 1 - 4 = -3$

And, $p(-2) = (-2)^2 - 4$

$= 4 - 4 = 0$

10. Verify whether the following are true or false.

(i) – 3 is a zero of x – 3.

(ii) $-\frac{1}{3}$ is a zero of 3x +1.

(iii) $\frac{-4}{5}$ is a zero of 4 – 5y.

(iv) 0 and 2 are the zeroes of $t^2 - 2t$.

(v) -3 is a zero of $y^2 + y - 6$.

Sol. A zero of a polynomial p(x) is a number c such that p(c) = 0

(i) Let $p(x) = x - 3$

$\therefore$ $p(-3) = -3 - 3 = -6 \neq 0$

Hence, - 3 is not a zero of x – 3.

(ii) Let $p(x) = 3x + 1$

$\therefore$ $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

Hence, $-\frac{1}{3}$ is zero of p(x) = 3x +1.

(iii) Let $p(y) = 4 - 5y$

$\therefore$ $p\left(-\frac{4}{5}\right) = 4 - 5\left(\frac{-4}{5}\right) = 4 + 4 = 8 \neq 0$

Hence, $-\frac{4}{5}$ is not a zero of 4 – 5y.

(iv) Let $p(t) = t^2 - 2t$

$\therefore$ $p(0) = (0)^2 - 2(0) = 0$

And $p(2)=(2)^2-2(2)=4-4=0$

Hence, 0 and 2 are zeroes of the polynomial $p(t)=t^2-2t.$

(v) Let $p(y)=y^2+y-6$

$\therefore \quad p(-3)=(-3)^2+(-3)-6=9-3-6=0$

Hence, - 3 is a zero of the polynomial $y^2+y-6.$

11. Find the zeroes of the polynomial in each of the following:

(i) $p(x)=x-4$

(ii) $g(x)=3-6x$

(iii) $q(x)=2x-7$

(iv) $h(y)=2y$

Sol. (i) Solving the equation $p(x)=0,$ we get

$x-4=0,$ which give us $x=4$

So, 4 is a zero of the polynomial $x-4.$

(ii) Solving the equation $g(x)=0,$ we get

$3-6x=0,$ which gives us $x=\frac{1}{2}$

SO, $\frac{1}{2}$ is a zero of the polynomial $3-6x.$

(iii) Solving the equation $q(x)=0,$ we get

$2x-7=0,$ which gives us $x=\frac{7}{2}$

So, $\frac{7}{2}$ is a zero of the polynomial $2x-7.$

(iv) Solving the equation $h(y)=0,$ we get

$2y=0,$ which gives us y = 0

So, 0 is a zero of the polynomial 2y.

12. Find the zeroes of the polynomial $(x-2)^2-(x+2)^2.$

Sol. Let $p(x)=(x-2)^2-(x+2)^2$

As fining a zero of p(x), is same as solving the equation p(x) = 0

So, $p(x)=0 \Rightarrow (x-2)^2-(x+2)^2=0$

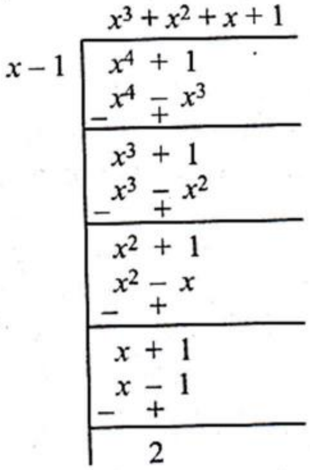
$\Rightarrow (x-2+x+2)(x-2-x-2)=0$

$\Rightarrow 2x(-4)=0 \Rightarrow -8x=0 \Rightarrow x=0$

Hence, x = 0 is the only one zero of p(x).

13. By acute division, find the quotient and the remainder when the first polynomial is divided by the second polynomial: $x^4+1; x+1.$

Sol. By acute division, we have



14. By remainder Theorem find the remainder, when p(x) is divided by g(x), where

(i) $p(x) = x^3 - 2x^2 - 4x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50, g(x) = x - 3$

(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3, g(x) = 2x - 1$

(iv) $p(x) = x^3 - 6x^2 + 2x - 4, g(x) = 1 - \frac{3}{2}x$

Sol. (i) We have $g(x) = x + 1$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -0$

Remainder = p (-1)

$= (-1)^3 - 2(-1)^2 - 4(-1) = -1 - 2 + 4 - 1$

= 0

(ii) We have $g(x) = x - 3$

$\Rightarrow \quad x - 3 = 0$

$\Rightarrow \quad x = 3$

Remainder = p(3)

$= (3)^3 - 3(3)^2 + 4(3) + 50 = 27 - 27 + 12 + 50$

= 62

(iii) We have $g(x) = 2x - 1$

$\Rightarrow \quad 2x - 1 = 0$

$\Rightarrow \quad 2x - 1 \Rightarrow x = \frac{1}{2}$

Remainder $= p\left(\frac{1}{2}\right)$

$$= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$

$$= 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 6 = \frac{1}{2} - 2 = \frac{-3}{2}$$

(iv) $g(x) = 0 \quad \Rightarrow 1 - \frac{3}{2}x = 0; x = \frac{2}{3}$

Remainder $p\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{24}{9} + \frac{4}{3} - 4$

$$= \frac{8 - 72 + 36 - 108}{27} = \frac{-136}{27}$$

15. Check whether p(x) is a multiple of g(x) or not:

(i) $p(x) = x^3 - 5x^2 + 4x - 3, g(x) = x - 2$

(ii) $p(x) = 2x^3 - 11x^2 - 4x + 5, g(x) = 2x + 1$

Sol. (i) p(x) will be a multiple g(x) if g(x) divides p(x).

Now, $g(x) = x - 2$ gives $x = 2$

Remainder $= p(2) = (2)^3 - 5(2)^2 + 4(2) - 3$

$$= 8 - 5(4) + 8 - 3 = 8 - 20 + 8 - 3$$

$$= -7$$

Since remainder $\neq 0$, So p(x) is not a multiple of g(x).

(ii) p(x) will be a multiple of g(x) if g(x) divides p(x).

Now, $g(x) = 2x + 1$ give $x = -\frac{1}{2}$

Remainder $= p\left(-\frac{1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 5$

$$= 2\left(\frac{-1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5 = \frac{-1}{4} - \frac{11}{4} + 7$$

$$= \frac{-1 - 11 + 28}{4} = \frac{16}{4} = 4$$

Since remainder $\neq 0$, So, p(x) is not a multiple of g(x).

16. Show that:

(i) x +3 is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

Sol. (i) Let $p(x) = 69 + 11x - x^2 + x^3, g(x) = x + 3$.

$g(x) = x + 3 = 0$ gives $x = -3$

g(x) will be a factor of p(x) if $p(-3)=0$ (Factor theorem)

Now, $p(-3)=69+11(-3)-(-3)^2+(-3)^3$

$=69-33-9-27$

$= 0$

Since, $p(-3)=0,$ So g(x) is a factor of p(x).

(ii) Let $p(x)=x+2x^3-9x^2+12$ and $g(x)=2x-3$

$g(x)=2x-3=0$ gives $x=\frac{3}{2}$

g(x) will be factor of p(x) if $p\left(\frac{3}{2}\right)=0$ (Factor theorem)

Now, $p\left(\frac{3}{2}\right)=\frac{3}{2}+2\left(\frac{3}{2}\right)^3-9\left(\frac{3}{2}\right)^2+12=\frac{3}{2}+2\left(\frac{27}{8}\right)-9\left(\frac{9}{4}\right)+12$

$=\frac{3}{2}+\frac{27}{4}-\frac{81}{4}+12=\frac{6+27-81+48}{4}=\frac{0}{4}=0$

Since, $p\left(\frac{3}{2}\right)=0,$ so, g(x) is a factor of p(x).

17. Determine which of the following polynomials has x– 2 a factor:

(i) $3x^2+6x-24$

(ii) $4x^2+x-2$

Sol. We know that if $(x-a)$ is a factor of p(x), then p(a) = 0.

(i) Let $P(x)=3x^2+6x-24$

If x – 2 is a factor of $p(x)=3x^2+6x-24,$ then p(2) should be equal to 0.

Now, $p(2)=3(2)^2+6(2)-24$

$=3(4)+6(2)-24$

= 12 + 12 – 24

= 0

$\therefore$ By factor theorem, $(x-2)$ is factor of $3x^2+6x-24.$

(ii) Let $p(x)=4x^2+x-2.$

If x – 2 is a factor of $p(x)=4x^2+x-2,$ then, p(2) should be equal to 0.

Now, $p(2)=4(2)^2+2-2$

$=4(4)+2-2$

= 16 + 2 – 2

$=16\neq 0$

$\therefore x-2$ is not a factor of $4x^2+x-2.$

18. Show that p – 1 is a factor of $p^{10}-1$ and also of $p^{11}-1$.

Sol. If $p-1$ is a factor of $p^{10}-1$, then $(1)^{10}-1$ should be equal to zero.

Now, $(1)^{10}-1=1-1=0$

Therefore, $p-1$ is a factor of $p^{10}-1$.

Again, if p – 1 is a factor of $p^{11}-1$, then $(1)^{11}-1$ should be equal to zero.

Now, $(1)^{11}-1=1-1=0$

Therefore, p – 1 is a factor of $p^{11}-1$.

Hence, p – 1 is a factor of $p^{10}-1$ and also of $p^{11}-1$.

19. For what value of m is x^3-2mx^2+16 divisible by $x+2$?

Sol. If x^3-2mx^2+16 is divisible by x + 2, then x + 2 is a factor of x^3-2mx^2+16.

Now, let $p(x)=x^3-2mx^2+16$.

As $x+2=x-(-2)$ is a factor of x^3-2mx^2+16.

So $p(-2)=0$

Now, $p(-2)=(-2)^3-2m(-2)^2+16$

$=-8-8m+16=8-8m$

Now, $p(-2)=0$

$\Rightarrow$ $8-8m=0$

$\Rightarrow$ $m=8\div 8$

$\Rightarrow$ $m=1$

Hence, for m + 1, x + 2 is a factor of $x^3-2mx^2+16,$ so x^3-2mx^2+16 is completely divisible by x +2.

20. If x + 2a is a factor of $x^5-4a^2x^3+2x+2a+3,$ find a.

Sol. Let $p(x)=x^5-4a^2x^3+2x+2a+3$

If $x-(-2a)$ is a factor of p(x), then $p(-2a)=0$

$\therefore$ $p(-2a)=(-2a)^5-4a^2(-2a)^3+2(-2a)+2a+3$

$=-32a^5+32a^5-4a+2a+3$

$=-2a+3$

Now, $p(-2a)=0$

$\Rightarrow$ $-2a+3=0$

$\Rightarrow$ $a=\frac{3}{2}$

21. Find the value of m so that 2x – 1 be a factor of $8x^4+4x^3-16x^2+10x+m.$

Sol. Let $p(x)=8x^4+4x^3-16x^2+10x+m.$

As $(2x-1)$ is a factor of p(x)

$\therefore \quad p\left(\frac{1}{2}\right)=0$ [By factor theorem]

$\Rightarrow \quad 8\left(\frac{1}{2}\right)^4+4\left(\frac{1}{2}\right)^3-16\left(\frac{1}{2}\right)^2+10\left(\frac{1}{2}\right)+m=0$

$\Rightarrow \quad 8\left(\frac{1}{16}\right)+4\left(\frac{1}{8}\right)-16\left(\frac{1}{4}\right)+5+m=0$

$\Rightarrow \quad \frac{1}{2}+\frac{1}{2}-4+5+m=0$

$\Rightarrow \quad 2+m=0 \Rightarrow m=-2$

22. If x + 1 is a factor of $ax^3+x^2-2x+4a-9,$ find the value of a.

Sol. Let $p(x)=ax^3+x^2-2x+4a-9.$

As (x + 1) is a factor of p(x)

$\therefore \quad p(-1)=0$ [By factor theorem]

$\Rightarrow \quad a(-1)^3+(-1)^2-2(-1)+4a-9=0$

$\Rightarrow \quad a(-1)+1+2+4a-9=0$

$\Rightarrow \quad -a+4a-6=0$

$\Rightarrow \quad 3a-6=0 \Rightarrow 3a=6 \Rightarrow a=2$

23. Factorise:

(i) $x^2+9x+18$

(ii) $6x^2+7x-3$

(iii) $2x^2-7x-15$

(iv) $84-2r-2r^2$

Sol. (i) In order to factorise $x^2+9x+18,$ we have to find two numbers p and q such that p + q = 9 and pq = 18.

Clearly, $6+3=9$ and $6\times 3=18.$

So, we write the middle term 9x as 6x + 3.

$\therefore \quad x^2+9x+18=x^2+6x+3x+18$

$=x(x+6)+3(x+6)$

$=(x+6)(x+3)$

(ii) In order to factorise $6x^2+7x-3,$ we have to find two numbers p and q such that $p+q=7$ and $pq=-18.$

Clearly, $9+(-2)=7$ and $9\times(-2)=-18.$

So, we write the middle term 7x as $9x+(-2x),$ i.e., $9x-2x.$

$\therefore \quad 6x^2+7x-3=6x^2+9x-2x-3$

$=3x(2x+3)-1(2x+3)$

$=(2x+3)(3x-1)$

(iii) In order to factorise $2x^3-7x-15$, we have to find two numbers p and q such that $p+q=-7$ and $pq=-30$.
Clearly, $(-10)+3=-7$ and $(-10)\times 3=-30$.
So, we write the middle term $-7x$ as $(-10x)+3x$.

$$\therefore 2x^2-7x-15=2x^2-10x+3x-15$$
$$=2x(x-5)+3(x-5)$$
$$(x-5)(2x+3)$$

(iv) In order to factorise $84-2r-2r^2$, we have to find two numbers p and 1 such that $p+q=-2$ and pq = - 168.

$$\therefore \quad 84-2r-2r^2=-2r^2-2r+84$$
$$=-2r^2-14r+12r+84$$
$$=-2r(r+7)+12(r+7)$$
$$=(r+7)(-2r+12)$$
$$=-2(r+7)(r-6)=-2(r-6)(r+7)$$

24. **Factorise:**
(i) $2x^3-3x^2-17x+30$
(ii) $x^3-6x^2+11x-6$
(iii) x^3+x^2-4x+4
(iv) $3x^3-x^2-3x+1$

Sol. (i) Let $f(x)=2x^3-3x^2-17x+30$ be the given polynomial. The factors of the constant term +30 are $\pm1,\pm2,\pm3,\pm5,\pm6,\pm10,\pm15,\pm30.$ The factor of coefficient of x^3 is 2. Hence, possible rational roots of f(x) are:

$$\pm1,\pm3,\pm5,\pm15,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{15}{2}.$$

We have $\quad f(2)=2(2)^3-3(2)^2-17(2)+30$
$$=2(8)-3(4)-17(2)+30$$
$$=16-12-34+30=0$$

And $\quad f(-3)=2(-3)^3-3(-3)^2-17(-3)+30$
$$=2(-27)-3(9)-17(-3)+30$$
$$=-54-27+51+30=0$$

So, $(x-2)$ and $(x+3)$ are factors of $f(x)$.
$\Rightarrow \quad x^2+x-6$ is a factor of $f(x)$.
Let us now divide $f(x)=2x^3-3x^2-17x+30$ by x^2+x-6 to get the other factors of $f(x)$.
Factors of $f(x)$.
By long division, we have

$$
\begin{array}{r|l|l}
x^2 + x - 6 & 2x^3 - 3x^2 - 17x + 30 & 2x - 5 \\
 & 2x^3 + 2x^2 - 12x & \\
 & - \quad - \quad + & \\
\hline
 & -5x^2 - 5x + 30 & \\
 & -5x^2 - 5x + 30 & \\
 & + \quad + \quad - & \\
\hline
 & 0 &
\end{array}
$$

$\therefore \quad 2x^3 - 3x^2 - 17x + 30 = (x^2 + x - 6)(2x - 5)$

$\Rightarrow \quad 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$

Hence, $2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$

(ii) Let $f(x) = x^3 - 6x^2 + 11x + 6$ be the given polynomial. The factors of the constant term – 6 are $\pm 1, \pm 2, \pm 3$ and ± 6.

We have, $\quad f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

And, $\quad f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 = 6 = 0$

So, $(x - 1)$ and $(x - 2)$ are factor of $f(x)$.

$\Rightarrow \quad (x - 1)(x - 2)$ is also factor of $f(x)$.

$\Rightarrow \quad x^3 - 3x + 2$ is a factor of $f(x)$.

Let us now divide $f(x) = x^3 - 6x^2 + 11x - 6$ by $x^2 - 3x + 2$ to get the other factors of $f(x)$.

By long division, we have

$$
\begin{array}{r|l|l}
x^2 - 3x + 2 & x^3 - 6x^2 + 11x - 6 & x - 3 \\
 & x^3 - 3x^2 + 2x & \\
 & - \quad + \quad - & \\
\hline
 & -3x^2 + 9x - 6 & \\
 & -3x^2 + 9x - 6 & \\
 & + \quad - \quad + & \\
\hline
 & 0 &
\end{array}
$$

$\therefore x^3 - 6x^2 + 11x - 6 = (x^2 - 3x + 2)(x - 3)$

$\Rightarrow x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

Hence, $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

(iii) Let $f(x) = x^3 + x^2 - 4x - 4$ be the given polynomial. The factors of the constant term -4 are $\pm 1, \pm 2, \pm 4$.

We have,

$\quad f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$

And, $\quad f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$

So, $\quad (x + 1)$ and $(x - 2)$ are factors of $f(x)$.

$\Rightarrow \quad (x + 1)(x - 2)$ is also a factor of $f(x)$.

$\Rightarrow \qquad x^2 - x - 2$ is a factor of $f(x)$.

Let us know divide $f(x) = x^3 + x^2 - 4x - 4$ by $x^2 - x - 2$ to get the other factors of $f(x)$.

By long division, we have

$$
\begin{array}{r|l|l}
x^2 - x - 2 & x^3 + x^2 - 4x - 4 & x + 2 \\
 & x^3 - x^2 + 2x & \\
 & \underline{-\quad +\quad +} & \\
 & 2x^2 - 2x - 4 & \\
 & 2x^2 - 2x - 4 & \\
 & \underline{-\quad +\quad +} & \\
 & 0 &
\end{array}
$$

$\therefore x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x + 2)$

$\Rightarrow x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$

Hence, $x^3 + x^2 - 4x - 4 = (x - 2)(x + 1)(x + 2)$

(iv) Let $f(x) = 3x^3 - x^2 - 3x + 1$ be the given polynomial. The factors of the constant term + 1 are ±1. The factor of coefficient of x^3 is 3. Hence, possible rational roots of $f(x)$ are: $\pm\frac{1}{3}$.

We have,

$$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1 = 3 - 1 - 3 + 1 = 0$$

And $\qquad f(-1) = 3(-1)^3 - (-1)^2 - 3(-1) + 1 = -3 - 1 + 3 + 1 = 0$

So, $(x - 1)$ and $(x + 1)$ are factors of $f(x)$.

$\Rightarrow (x - 1)(x + 1)$ is also a factor of $f(x)$.

$\Rightarrow x^2 - 1$ is a factor of $f(x)$.

Let us now divide $f(x) = 3x^3 - x^2 - 3x + 1$ by $x^2 - 1$ to get the other factors of $f(x)$.

By long division, we have

$$
\begin{array}{r|l|l}
x^2 - 1 & 3x^3 - x^2 - 3x + 1 & 3x - 1 \\
 & 3x^3 \quad - 3x & \\
 & \underline{-\qquad +} & \\
 & -x^2 + 1 & \\
 & -x^2 + 1 & \\
 & \underline{+\quad -} & \\
 & 0 &
\end{array}
$$

$\therefore \qquad 3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$

$\Rightarrow \qquad 3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

Hence, $3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

25. Using suitable identify, evaluate the following:

(i) 103^3

(ii) 101×102

(iii) 999^2

Sol. (i) $103^2 = (100+3)^3$

Now using identify $(a+b)^3 = a^3+b^3+3ab(a+b)$, we have

$(100+3)^3 = (100)^3+(3)^3+3(100)(3)(100+3)$

= 1000000 + 27 + 900(100 + 3)

= 1000000 + 27 + 90000 + 2700

= 1092727

(ii) 101×102 = (100 + 1) (100 + 2)

Now, using identify $(x+a)(x+b) = x^2+(a+b)x+ab$, we have

$(100+1)(100+2) = (100)^2+(1+2)100+(1)(2)$

= 10000 + (3)100 + 2 = 10000 + 300 + 2

= 10302

(iii) $(999)^2 = (1000-1)^2 = (1000)^2 - 2\times(1000)\times1+1^2$

= 100000 – 2000 + 1

= 998001

26. Factorise the following:

(i) $4x^2+20x+25$

(ii) $9y^2-66yz+121z^2$

(iii) $\left(2x+\frac{1}{3}\right)^2-\left(x-\frac{1}{2}\right)^2$

Sol. (i) We have,

$4x^2+20x+25 = (2x)^2+2(2x)(5)+(5)^2$

$= (2x+5)^2 \quad [\because a^2+2ab+b^2 = (a+b)^2]$

$= (2x+5)(2x+5)$

(ii) We have,

$9y^2-66yz+121z^2 = (-3y)^2+2(-3y)(11z)+(11z)^2$

$= (-3y+11z)^2 \quad [\because a^2+2ab+b^2 = (a+b)^2]$

$= (-3y+11z)(-3y+11z)$

$= (3y-11z)(3y-11z)$

(iii) $\left(2x+\frac{1}{3}\right)^2-\left(x-\frac{1}{2}\right)^2$

Using identity $a^2-b^2 = (a+b)(a-b)$

$= \left[\left(2x+\frac{1}{3}\right)+\left(x-\frac{1}{2}\right)\right]\left[\left(2x+\frac{1}{3}\right)-\left(x-\frac{1}{2}\right)\right]$

$$=\left(2x+\frac{1}{3}+x-\frac{1}{2}\right)\left(2x+\frac{1}{3}-x+\frac{1}{2}\right)=\left(3x-\frac{1}{6}\right)\left(x+\frac{5}{6}\right)$$

27. **Factorise the following:**

(i) $9x^2-12x+3$

(ii) $9x^2-12x+4$

Sol. (i) $9x^2-12x+3=9x^2-9x-3x+3$

$=9x(x-1)-3(x-1)$

$=(9x-3)(x-1)$

$=3(3x-1)(x-1)$

(ii) We have,

$9x^2-12x+4=(3x)^2-2(3x)(2)+(2)^2$

$=(3x-2)^2$ $[\because a^2-2ab+b^2=(a-b)^2]$

$=(3x-2)(3x-2)$

28. **Expand the following:**

(i) $(4a-b+2c)^2$

(ii) $(3a-5b-c)^2$

(iii) $(-x+2y-3z)^2$

Sol. (i) We have,

$(4a-b+2c)^2=(4a)^2+(-b)^2+(2c)^2+2(4a)(-b)+2(-b)(2c)+2(2c)(4a)$

$\left[\because a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2\right]$

$=16a^2+b^2+4c^2-8ab-4ac+16ca$

(ii) We have,

$(3a-5b-c)^2=(3a)^2+(-5b)^2+(-c)^2+2(3a)^2-5b+2(-5b)(-c)+2(-c)(3a)$

$\left[\because a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2\right]$

$=9a^2+25b^2+c^2-30ab+10bc-6ca.$

(iii) $(-x+2y-3z)^2=\{(-x)+2y+(-3z)\}^2$

$=(-x)^2+(2y)^2+(-3z)^2+2(-x)(2y)+2(2y)(-3z)+2(-3yz)(-x)$

29. **Factorise the following:**

(i) $9x^2+4y^2+16z^2+12xy-16yz-24xz$

(ii) $25x^2+16y^2+4z^2-40xy+16yz-20xz$

(iii) $16x^2+4y^2+9z^2-16xy-12yz+24xz$

Sol. (i) We have,

$9x^2+4y^2+16z^2+12xy-16yz-24xz$

$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$

$= \{3x + 2y + (-4z)\}^2 \left[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right]$

$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$

(ii) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

$= (-5x)^2 + (4y)^2 + (2z)^2 + 2.(-5x)(4y) + 2(4y)(2z) + 2(2z)(-5x)$

$= (-5x + 4y + 2z)^2$

(ii) We have,

$16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

$= (4x)^2 + (-2y)^2 + (3z)^2 + 2(4x)(-2y) + 2(-2y)(3z) + 2(3z)(4x)$

$= \{4x + (-2y) + 3z\}^2 \left[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right]$

$= (4x - 2y + 3z)^2$

$= (4x - 2y + 3z)(4x - 2y + 3z)$

30. If $a + b + c = 9$ **and** $ab + bc + ca = 26,$ **find** $a^2 + b^2 + c^2.$

Sol. We have that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + bc + 2ca$

$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(26)$

[Putting the value of $a + b + c$ and $ab + bc + ca$]

$\Rightarrow 81 = (a^2 + b^2 + c^2) + 52$

$\Rightarrow (a^2 + b^2 + c^2) = 81 - 52 = 29$

31. Expand the following:

(i) $(3a - 2b)^3$

(ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

(iii) $\left(4 - \frac{1}{3x}\right)^3$

Sol. (i) We have

$(3a - 2b)^3 = (3a)^3 - (2b)^3 - 3(3a)(2b)(3a - 2b)$

$\left[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right]$

$= 27a^3 - 8b^3 - 18ab(3a - 2b)$

$= 27a^3 - 8b^3 - 54a^2b + 36ab^2$

(ii) $\because \quad (x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$$\therefore \quad \left(\frac{1}{x}+\frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\times\frac{1}{x}\times\frac{y}{3}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$= \frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$= \frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x^2}+\frac{y^3}{3x} = \frac{1}{x^3}+\frac{y}{x^2}+\frac{y^2}{3x}+\frac{y^3}{27}$$

(iii) We have,

$$\left(4-\frac{1}{3x}\right)^3 = (4)^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4-\frac{1}{3x}\right)$$

$$\left[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right]$$

$$= 64 - \frac{1}{27x^3} - \frac{4}{x}\left(4-\frac{1}{3x}\right)$$

$$= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}$$

32. Factorise the following:

(i) $1-64a^3-12a+48a^2$

(ii) $8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

Sol. (i) We have,

$$1-64a^3-12a+48a^2 = (1)^3-(4a)^3-3(1)(4a)(1-4a)$$

$$= (1-4a)^3 \left[\because a^3-b^3-3ab(a-b) = (a-b)^3\right]$$

$$= (1-4a)(1-4a)(1-4a)$$

(ii) $8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

$$= (2p)^3 + 3\times(2p)^2\times\frac{1}{5} + 3\times(2p) + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3$$

$$= (2p)^3 + \left(\frac{1}{5}\right)^3 + 3\times(2p)\times\frac{1}{5}\left[2p+\frac{1}{5}\right]$$

Now, using $a^3+b^3+3ab(a+b) = (a+b)^3$

$$= \left(2p+\frac{1}{5}\right)^3 = \left(2p+\frac{1}{5}\right)\left(2p+\frac{1}{5}\right)\left(2p+\frac{1}{5}\right)$$

33. Find the following produces:

(i) $\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$

(ii) $(x^2-1)(x^4+x^2+1)$

Sol. (i) We have,

$$\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)=\left(\frac{x}{y}+2y\right)\left\{\left(\frac{x}{2}\right)^2-\left(\frac{x}{2}\right)(2y)+(2y)^2\right\}$$

$$=\left(\frac{x}{2}\right)^3+(2y)^3 \qquad \left[\because (a+b)(a^2-ab+b^2)=a^3+b^3\right]$$

$$=\frac{x^3}{8}+8y^3$$

(ii) We have,

$$(x^2-1)(x^4+x^2+1)=(x^2-1)\{(x^2)^2+(x^2)(1)+(1)^2\}$$

$$=(x^2)^3-(1)^3$$

$$[\because (a-b)(a^2+ab+b)^2=a^3-b^3]$$

$$=x^6-1$$

34. Factorise:

(i) $1+64x^3$

(ii) $a^3-2\sqrt{2b^3}$

Sol. (i) We have,

$$1+64x^3=(1)^3+(4x)^3$$

$$=(1+4x)\{(1)^2-(1)(4x)+(4x)^2\}$$

$$[\because a^3+b^3=(a+b)(a^2-ab+b^2)]$$

$$=(1+4x)(1-4x+16x^2)$$

$$=(1+4x)(16x^2-4x+1)$$

$$=(4x+1)(16x^2-4x+1)$$

(ii) We have,

$$a^3-2\sqrt{2b^3}=(a)^3-(\sqrt{2}b)^3$$

$$=(a-\sqrt{2}b)\{(a)^2+(a)(\sqrt{2}b)+(\sqrt{2}b)^2\}$$

$$\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$=(a-\sqrt{2}b)(a^2+\sqrt{2}ab+2b^2)$$

35. Find the following product:

$(2x-y+3z)(4x^2+y^2+9z^2+2xy+3yz-6xz)$

Sol. We have,

$(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

$= \{2x + (-y) + 3z\}\{(2x)^2 + (-y)^2 + (3z)^2 - (2x)(-y) - (-y)(3z) - (3z)(2x)\}$

$= (2x)^3 + (-y)^3 + (3z)^3 - 3(2x)(-y)(3z)$

$[\because (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$

$= 8x^3 - y^3 + 27z^2 + 18xyz$

36. Factorise:

(i) $a^3 - 8b^3 - 64c^3 - 24abc$

(ii) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

Sol. (i) We have,

$a^3 - 8b^3 - 64c^3 - 24abc$

$= \{(a)^3 + (-2b)^3 + (-4c)^3 - 3(a)(-2b)(-4c)\}$

$= \{a + (-2b) + (-4c)\}\{a^2 + (-2b)^2 + (-4c)^2 - a(-2b) - (-2b)(-4c) - (-4c)a\}$

$[\because a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= (a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$

(ii) We have,

$2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

$= \{(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)(2b)(-3c)\}$

$= \{\sqrt{2}a + 2b + (-3c)\}\{(\sqrt{2}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{2}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{2}a)\}$

$= (\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ca)$

37. Without actually calculating the cubes, find the value of:

(i) $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

(ii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

Sol. (i) Let $a = \frac{1}{2}, b = \frac{1}{3}, c = -\frac{5}{6}$

$\therefore \quad a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$

$= \frac{3 + 2 - 5}{6} = \frac{0}{6} = 0$

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

$\therefore \quad \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$

$$= 3 \times \frac{1}{2} \times \frac{1}{3}\left(-\frac{5}{6}\right) = -\frac{5}{12}$$

(ii) We have,

$(0.2)^3 - (0.3)^3 + (0.1)^3 = (0.2)^3 + (-0.3)^3 + (0.1)^3$

Let $a = 0.2, b = -0.3$ and $c = 0.1$. Then,

$$a + b + c = 0.2 + (-0.3) + 0.1$$
$$= 0.2 - 0.3 + 0.1 = 0$$

$\because \quad a + b + c = 0$

$\because \quad a^3 + b^3 + c^3 = 0 = 3abc$

$\Rightarrow \quad (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3(0.2)(-0.3)(0.1) = -0.018$

Hence, $(0.2)^3 + (-0.3)^3 + (0.1)^3 = -0.018$

38. Without finding the cubes, factorise

$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

Sol. **Let** $x - 2y = a, 2y - 3z = b$ and $3z - c = x$

$\therefore \quad a + b + c = x - 2y + 2y - 3z + 3z - x = 0$

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

Hence, $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

$$= 3(x - 2y)(2y - 3z)(3z - x)$$

39. Find the value of

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

(ii) $x^3 + 8y^3 - 36xy - 216$, when $x = 2y + 6$

Sol. (i) $x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3x \times y \times 4$

$$= (x + y + 4)(x^2 + y^2 + 4^2 - xy - 4y - 4x)$$

$$[\because x + y = -4]$$

$$= (0)(x^2 + y^2 + 4^2 - xy - 4y - 4x) = 0$$

(ii) $x^3 + 8y^3 - 36xy - 216 = x^3 + (-2y)^3 + (-6)^3 - 3x(-2y)(-6)$

$$= (x - 2y - 6)$$
$$[x^2 + (-2y)^2 + (-6)^2 - x(-2y) - (-2y)(-6) - (-6)x]$$
$$= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$$
$$= (0)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) = 0$$

$$[\because x = 2y + 6]$$

40. Give possible experiments for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

Sol. Area: $4a^2 + 4a - 3$.

Using the method of splitting the middle term, we first two numbers whose sum is +4 and produce is $4\times(-3)=-12$.

Now, $+6-2=+4$ and $(+6)\times(-2)=-12$

We split the middle term 4a as $4a=+6a-2a$,

So, that $4a+4a-3=4a^2+6a-2a-3$

$=2a(2a+3)-1(2a+3)$

$=(2a-1)(2a+3)$

Now, area of rectangle $=4a^2+4a-3$

Also, area of rectangle = length × breadth and $4a^2+4a-3=(2a-1)(2a+3)$

So, the possible expressions for the length and breadth of the rectangle are length $=(2a-1)$ and breadth $=(2a+3)$ or, length $=(2a+3)$ and breadth $=(2a-1)$.

Polynomials
<u>Exercise 2.4</u>

1. **If the polynomials** az^3+4z^2+3z-4 **and** z^3-4z+a **leave the same remainder when divided by** $z-3$, **Find the value of a.**

Sol. Let $p(z)=az^3+4z^2+3z-4$

And $q(z)=z^3-4z+a$

As these two polynomials leave the same remainder, when divided by $z-3$, then p(3) = q(3).

$\therefore \quad p(3)=a(3)^3+4(3)^2+3(3)-4$

$=27a+36+9-4$

Or $p(3)=27a+41$

And $q(3)=(3)^3-4(3)+a$

$=27-12+a=15+a$

Now, $p(3)=q(3)$

$\Rightarrow \quad 27a+41=15+a$

$\Rightarrow \quad 26a=-26a; a=-1$

Hence, the required value of $a=-1$.

2. **The polynomial** $p(x)=x^4-2x^3+3x^2-ax+3a-7$ **when divided by** $x+1$ **leave remainder 19. Also, find the remainder when** $p(x)$ **is divided by** $x+2$.

Sol. We know that if $p(x)$ is divided by $x+a$, then the remainder $=p(-a)$.

Now, $p(x)=x^4-2x^3+3x^3-ax+3a-7$ is divided by $x+1$, then the remainder $=p(-1)$

Now, $p(-1)=(-1)^4-2(-1)^3+3(-1)^2-a(-1)+3a-7$

$=1-2(-1)+3(1)+a+3a-7$

$=1+2+3+4a-7$

$=-1+4a$

Also, remainder = 19

$\therefore \quad -1+4a=19$

$\Rightarrow \quad 4a=20; a=20\div 4=5$

Again, when p(x) is divided by $x+2$, then

Remainder $=p(-2)=(-2)^4-2(-2)^3+3(-2)^2-a(-2)+3a-7$

$=16+16+12+2a+3a-7$

$=37+5a$

$=37+5(5)=37+25=62$

3. **If both** $(x-2)$ **and** $\left(x-\frac{1}{2}\right)$ **are factors of** px^2+5x+r, **Show that p = r.**

Sol. Let $p(x)=px^2+5x+r$.

As $(x-2)$ is a factor of p(x)

So, $p(2)=0 \Rightarrow P(2)^2+5(2)+r=0$

$\Rightarrow \quad 4p+10+r=0 \quad \ldots(1)$

Again, $\left(x-\frac{1}{2}\right)$ is factor of $p(x)$.

$\therefore \quad p\left(\frac{1}{2}\right)=0$

Now, $p\left(\frac{1}{2}\right)=p\left(\frac{1}{2}\right)^2+5\left(\frac{1}{2}\right)+r$

$=\frac{1}{4}p+\frac{5}{2}+r$

$\therefore \quad p\left(\frac{1}{2}\right)=0 \Rightarrow \frac{1}{4}p+\frac{5}{2}+r=0 \quad \ldots(2)$

From (1), we have $4p+r=-10$

From (2), we have $p+10+4r=0$

$\Rightarrow \quad p+4r=-10$

$\therefore \quad 4p+r=p+4r \quad [\because \text{Each} = -10]$

$\therefore \quad 3p=3r \Rightarrow p=r$

Hence, proved.

4. Without actual division, prove that $2x^4-5x^3+2x^2-x+2$ **is divisible by** x^2-3x+2.

Sol. We have,

$$x^2-3x+2=x^2-x-2x+2$$
$$=x(x-1)-2(x-1)$$
$$=(x-1)(x-2)$$

Let $p(x)=2x^4-5x^3+2x^2-x+2$

Now, $p(1)=2(1)^4-5(1)^3+2(1)^2-1+2=2-5+2-1+2=0$

Therefore, (x – 1) divides p(x)

And $p(2)=2(2)^4-5(2)^3+2(2)^2-2+2$

= 32 – 40 + 8 – 2 + 2 = 0

Therefore, $(x-2)$ divides p(x).

So, $(x-1)(x-2)=x^2-3x+2$ divides $2x^4-5x^3+2x^2-x+2$

5. Simplify $(2x-5y)^3-(2x+5y)^3$.

Sol. We have,

$(2x-5y)^3-(2x+5y)^3$

$=\{(2x-5y)-(2x+5y)\}\{(2x-5y)^2+(2x-5y)(2x+5y)+(2x+5y)^2\}$

$\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$= (2x - 5y - 2x - 5y)\ (4x^2 + 25y^2 - 20xy + 4x^2 - 25y^2 + 4x^2 + 25y^2 + 20xy)$
$= (-10y)(2x^2 + 25y^2)$
$= -120x^2y - 250y^3$

6. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

Sol. We have,

$(-z + x - 2y)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz)$
$= \{(x + (-2y) + (-z)\}\ \{(x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (-2y)(-z) - (-z)(x)\}$
$= x^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z)$
$[\because (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc\ ]$
$= x^3 - 8y^3 - z^3 - 6xyz$

7. If a, b, c are all non-zero and a + b + c = 0, prove that
$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3.$

Sol. We have a, b, c are all non-zero and a + b + c = 0, therefore

$a^3 + b^3 + c^3 = 3abc$

Now, $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$

8. If a + b + c = 5 and ab + bc + ca = 10, then prove that $a^3 + b^3 + c^3 - 3abc = -25$

Sol. We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$
$= (a + b + c)\ [a^2 + b^2 + c^2 - (ab + bc + ca)]$
$= 5\{a^2 + b^2 + c^2 - (ab + bc + ca)\}$
$= 5(a^2 + b^2 + c^2 - 10)$

Now, a + b + c = 5
Squaring both sides, we get

$(a + b + c)^2 = 5^2$
$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$
$\therefore \quad a^2 + b^2 + c^2 + 2(10) = 25$
$\Rightarrow \quad a^2 + b^2 + c^2 = 25 - 20 = 5$

Now, $a^3 + b^3 + c^3 - 3abc = 5(a^2 + b^2 + c^2 - 10)$
$= 5(5 - 10) = 5(-5) = -25$

Hence, proved.

9. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

Sol. $(a+b+c)^3 = [a+(b+c)]^3$

$$= a^3 + 3a^2(b+c) + 3a(b+c)^2 + (b+c)^3$$
$$= a^3 + 3a^2b + 3a^2c + 3a\ (b^2 + 2bc + c^2) + (b^3 + 3b^2c + 3bc^2 + c^3)$$
$$= a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$$
$$= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3b^2a + 3c^2a + 3c^2b + 6abc$$
$$= a^3 + b^3 + c^3 + 3a^2(b+c) + = a^3 + b^3 + c^3 + 3a^2(b+c) +$$

Hence, above result can be put in the form

$$(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$$

$$\therefore (a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$$