Exercise 12.1 : Solutions of Questions on Page Number : 197

Q1 :

Evaluate

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Answer :

(i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

(iii) $\left(\frac{1}{2}\right)^{-5} = \frac{1}{(2)^{-5}} = (2)^5 = 2\times2\times2\times2\times2 = 32$

Q2 :

Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Answer :

(i) $(-4)^5$ ÃƒÂ· $(-4)^8 = (-4)^{5-8}$ (a^m ÃƒÂ· $a^n = a^{m-n}$)
$= (-4)^{-3}$

$$= \frac{1}{(-4)^3} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$\left(\frac{1}{2^3}\right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^6} \qquad \left((a^m)^n = a^{mn}\right)$$

(ii)

$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4}$$

(iii)

$$= (-1)^4 \times 3^4 \times \frac{5^4}{3^4} \qquad \left[(ab)^m = a^m \times b^m\right]$$
$$= (-1)^4 \times 5^4$$
$$= 5^4 \qquad \left[(-1)^4 = 1\right]$$

(iv) $(3^{-7}$ ÃƒÂ· $3^{-10}) \times 3^{-5} = (3^{-7-(-10)}) \times 3^{-5}$ (a_m ÃƒÂ· $a_n = a_{m-n}$)

$= 3^3 \times 3^{-5}$

$= 3^{3+(-5)}$ ($a_m \times a_n = a_{m+n}$)

$= 3^{-2}$

$$= \frac{1}{3^2} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$$

(v) $2^{-3} \times (-7)^{-3} =$ $\dfrac{1}{2^3} \times \dfrac{1}{(-7)^3}$ $\qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$$= \frac{1}{\left[2 \times (-7)\right]^3} \qquad \left[a^m \times b^m = (ab)^m\right]$$
$$= \frac{1}{(-14)^3}$$

Q3 :
Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1})$ ÃƒÂ·2^{-2}

$$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$$

((v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$ iii) (iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

Answer :

(i) $(3^0+4^{-1})\times 2^2=\left(1+\frac{1}{4}\right)\times 2^2$ $\left(a^0=1 \text{ and } a^{-m}=\frac{1}{a^m}\right)$

$$=\frac{5}{4}\times 4=5$$

(ii) $(2^{-1} \times 4^{-1})$ ÃƒÂ· $2^{-2} = [2^{-1} \times \{(2)_2\}^{-1}]$ ÃƒÂ· 2^{-2}

$= (2^{-1} \times 2^{-2})$ ÃƒÂ· 2^{-2} $\left(\left(a^m\right)^n=a^{mn}\right)$

$= 2^{-1+(-2)}$ ÃƒÂ· 2^{-2} ($a_m \times a_n = a_{m+n}$)

$= 2^{-3}$ ÃƒÂ· 2^{-2}

$= 2^{-3-(-2)}$ (a_m ÃƒÂ· $a_n = a_{m-n}$)

$= 2^{-3+2} = 2^{-1}$

$= \frac{1}{2}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = \left(\frac{2}{1}\right)^{2} + \left(\frac{3}{1}\right)^{2} + \left(\frac{4}{1}\right)^{2}$ $\left(\therefore a^{-m} = \frac{1}{a^m}\right)$

$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0$ $\left(a^{-m} = \frac{1}{a^m}\right)$

= 1 ($a^0 = 1$)

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left\{\left(\frac{3}{-2}\right)^{2}\right\}^2$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \left\{\frac{3^2}{(-2)^2}\right\}^2$ $\left[\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= \left(\frac{9}{4}\right)^2 = \frac{81}{16}$

Q4 :

Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $\left(5^{-1} \times 2^{-1}\right) \times 6^{-1}$

Answer :

(i) $\frac{8^{-1}\times 5^3}{2^{-4}} = \frac{2^4\times 5^3}{8^1} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{2^4\times 5^3}{2^3} = 2^{4-3}\times 5^3 \qquad \left(a^m \div a^n = a^{m-n}\right)$

$= 2\times 125 = 250$

(ii) $(5^{-1}\times 2^{-1})\times 6^{-1} = \left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{10}\times\frac{1}{6} = \frac{1}{60}$

Q5 :
Find the value of *m* for which 5^m ÃƒÂ·$5^{-3} = 5^5$.

Answer :

5^m ÃƒÂ· $5^{-3} = 5^5$

$5^{m-(-3)} = 5^5$ (a_m ÃƒÂ· $a_n = a_{m-n}$)

$5^{m+3} = 5^5$

Since the powers have same bases on both sides, their respective exponents must be equal.

$m + 3 = 5$

$m = 5 - 3$ m

$= 2$

Q6 :

Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Answer :

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}^{-1}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$=\{3-4\}^{-1}=(-1)^{-1}=\frac{1}{-1}=-1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$ $\left[\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right]$

$=\frac{8^{7}}{5^{7}}\times\frac{5^{4}}{8^{4}}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$=\frac{8^{7-4}}{5^{7-4}}$ $\left(a^{m}\div a^{n}=a^{m-n}\right)$

$=\frac{8^{3}}{5^{3}}=\frac{512}{125}$

Q7 :

Simplify. (i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}(t\neq 0)$ (ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Answer :

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}=\frac{5^2\times t^{-4}}{5^{-3}\times 5\times 2\times t^{-8}}$

$=\frac{5^2\times t^{-4}}{5^{-3+1}\times 2\times t^{-8}} \qquad \left(a^m\times a^n=a^{m+n}\right)$

$=\frac{5^2\times t^{-4}}{5^{-2}\times 2\times t^{-8}}$

$=\frac{5^{2-(-2)}t^{-4-(-8)}}{2} \qquad \left(a^m\div a^n=a^{m-n}\right)$

$=\frac{5^4t^4}{2}=\frac{625\,t^4}{2}$

(ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^3}{5^{-7}\times(2\times 3)^{-5}}$

$=\frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^3}{5^{-7}\times 2^{-5}\times 3^{-5}} \qquad \left[(a\times b)^m=a^m\times b^m\right]$

$=3^{-5-(-5)}\times 2^{-5-(-5)}\times 5^{-5+3-(-7)} \qquad \left(a^m\div a^n=a^{m-n}\right)$

$=3^0\times 2^0\times 5^5 \qquad \left(a^0=1\right)$

$=5^5$

Exercise 12.2 : Solutions of Questions on Page Number : 200

Q1 :

Express the following numbers in standard form.

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

Answer :

(i) 0.0000000000085 = 8.5 x 10^{-12}

(ii) $0.00000000000942 = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

(v) $31860000000 = 3.186 \times 10^{10}$

Q2 :

Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^{4}

(iii) 3×10^{-8} (iv) 1.0001×10^{9}

(v) 5.8×10^{12} (vi) 3.61492×10^{6}

Answer :

(i) $3.02 \times 10^{-6} = 0.00000302$

(ii) $4.5 \times 10^{4} = 45000$
(iii) $3 \times 10^{-8} = 0.00000003$

(iv) $1.0001 \times 10^{9} = 1000100000$

(v) $5.8 \times 10^{12} = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3614920$

Q3 :

Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer :

(i) $\frac{1}{1000000} = 1 \times 10^{-6}$

(ii) 0.000, 000, 000, 000, 000, 000, 16 = 1.6×10^{-19}

(iii) 0.0000005 = 5×10^{-7}

(iv) 0.00001275 = 1.275×10^{-5}

(v) 0.07 = 7×10^{-2}

Q4 :
In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer :

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5 x 20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5 x 0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

= (100 + 0.080) mm

= 100.08 mm

= 1.0008×10^2 mm