Unit 11(Mensuration)

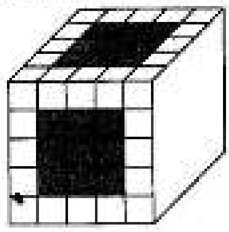
Multiple Choice Questions

Question. 1 A cube of side 5 cm is painted on all its faces. If it is sliced into 1 cubic centimetre cubes, then how many 1 cubic centimetre cubes will have exactly one of their faces painted?

(a) 27 (b) 42 (c) 54 (d) 142

Solution.

(c) Given, a cube of side 5 cm is painted on all its faces and is sliced into 1 cm^3 cubes. Then, from figure, it is clear that there are 9 cubes available on face.



Since, there are six faces available.

Hence, total number of smaller cubes = $6 \times 9 = 54$

Question. 2 A cube of side 4 cm is cut into 1 cm cubes. What is the ratio of the surface areas of the original cube and cut-out cubes?

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 6

Solution.

(c) Volume of the original cube having side of length 4 cm = $(4)^3 = 64$ cm^3

[$\because$ volume of cube with side $a = a^3$]

Volume of the cut-out cubes with side of length 1 cm = 1 cm^3

$\therefore$ Number of cut-out cubes = $\frac{\text{Volume of the original cube}}{\text{Volume of a smaller cube}} = \frac{64}{1} = 64$

Now, surface area of cut-out cubes = $64 \times 6 \times (1)^2$ cm^2

[$\because$ surface area of cube with side $a = 6a^2$]

and surface area of the original cube = 6×4^2 cm^2

$\therefore$ The required ratio of surface areas of the original cube and cut-out cubes

$$= \frac{6 \times 4^2}{64 \times 6} = 1 : 4$$

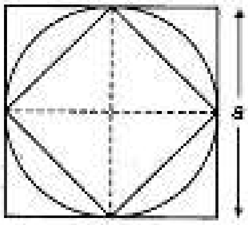
Question. 3 A circle of maximum possible size is cut from a square sheet of board. Subsequently, a square of maximum possible size is cut from the resultant circle. What will be the area of the final square?

(a) 3/4 of original square (b) 1/2 of original square

(c) 1/4 of original square (d) 2/3 of original square

Solution.

(b) Let a be the side of a square sheet.



Then, area of bigger square sheet $= a^2$ ...(i)

Now, we make the circle of maximum possible size from it.

Then, the radius of circle $= \frac{a}{2}$ ...(ii)

So, its diameter $(d') = 2 \cdot \frac{a}{2} = a$

Now, any square in a circle of maximum size will have the length of diagonal equal to the diameter of circle.

i.e. diagonal of square made inside the circle $= a$

So, the side of this square $= \frac{a}{\sqrt{2}}$ [$\because$ diagonal $=$ side $\sqrt{2}$]

$\therefore$ Area of this square $= a^2/2$ ...(iii)

From Eqs. (i) and (iii),

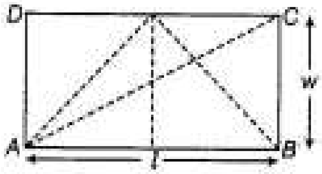
Area of final square is $\frac{1}{2}$ of original square.

Question.4 What is the area (in sq units) of the largest triangle, that can be fitted into a rectangle of length l units and width w units?

(a)lw/2 (b) lw/3 (c) lw/6 (d) lw/4

Solution.

(a)



Let $ABCD$ be the rectangle of length l and width w.

Now, we construct a triangle of maximum area inside it in all possible ways.

$\because$ We know that,

Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

So, for maximum area, base and height of maximum, length is needed.

Here, maximum base length $= l$

and maximum height $= w$

$\therefore$ Area (maximum) of triangle $= \frac{1}{2} \times l \times w = \frac{l \times w}{2}$ sq units

Question. 5 If the height of a cylinder becomes1/4 of the original height and the radius is doubled, then which of the following will be true?

(a) Volume of the cylinder will be doubled

(b) Volume of the cylinder will remain unchanged

(c) Volume of the cylinder will be halved

(d) Volume of the cylinder will be 1/4 of the original volume

Solution.

(b) We know that, the volume of a cylinder having base radius r and height h is $V = \pi r^2 h$

Now, if new height is $\frac{1}{4}$th of the original height and the radius is doubled, i.e.

$h' = \frac{1}{4}h$ and $r' = 2r$, then

New volume, $V' = \pi (2r)^2 \times \frac{1}{4}h = 4\pi r^2 \times \frac{1}{4}h$

$= \pi r^2 h = V$

Hence, the new volume of cylinder is same as the original volume.

Question. 6 If the height of a cylinder becomes 1/4 of the original height and the radius is doubled, then which of the following will be true?

(a) Curved surface area of the cylinder will be doubled

(b) Curved surface area of the cylinder will remain unchanged

(c) Curved surface area of the cylinder will be halved

(d) Curved surface area will be 1/4 of the original curved surface

Solution.

(c) Let the new height and radius be $\frac{h}{4}$ and $2r$ respectively, where r and h are original radius and original height respectively of the cylinder.

We know that, curved surface area of cylinder $= 2\pi rh$

Then, curved surface of the new cylinder

$= 2\pi (2r) \times \frac{1}{4}h = 4\pi r \times \frac{1}{4}h = \pi rh$

$= \frac{1}{2} \times 2\pi rh$ [multiplying and dividing by 2]

$= \frac{1}{2}$ original curved surface area

Hence, the curved surface area of the cylinder will be halved.

Question. 7 If the height of a cylinder becomes 1/4 of the original height and the radius is doubled, then which of the following will be true?

(a) Total surface area of the cylinder will be doubled

(b) Total surface area of the cylinder will remain unchanged

(c) Total surface area of the cylinder will be halved
(d) None of the above
Solution.
(d) We know that,

Total surface area of cylinder having radius r and height $h = 2\pi r(h + r)$

Total surface area of the cylinder with new height $\left(\frac{h}{u}\right)$ and radius $2r$

$$= 2\pi(2r)\left(2r + \frac{1}{4}h\right)$$

$$\therefore = 4\pi r(8r + h) \times \frac{1}{4}$$

$$= \pi r(8r + h)$$

Question. 8 The surface area of the three coterminus faces of a cuboid are 6, 15 and 10 cm2, respectively. The volume of the cuboid is
(a) 30 Cm^3 (b) 40 Cm^3 (c) 20 Cm^3 (d) 35 Cm^3
Solution.
(a) If l, b and h are the dimensions of the cuboid. Then,

Volume of the cuboid $= l \times b \times h$

Here, $6 = l \times b$

$15 = l \times h$

$10 = b \times h$

$\therefore$ $6 \times 15 \times 10 = l^2b^2h^2$

$\therefore$ Volume $= l \times b \times h$

$= \sqrt{6 \times 15 \times 10} = 30\text{cm}^3$

Question. 9 A regular hexagon is inscribed in a Circle of radius r. The perimeter of the regular hexagon is
(a) 3r (b) 6r (c)9r (d)12r
Solution.
(b) A regular hexagon comprises 6 equilateral triangles, each of them having one of their vertices at the centre of the hexagon.
The sides of the equilateral triangle are equal to the radius of the smallest circle inscribing the hexagon.
Hence, each side of the hexagon is equal to the radius of the hexagon and the perimeter of the hexagon is 6 r.

Question. 10 The dimensions of a godown are 40 m, 25 m and 10 m. If it is filled with . cuboidal boxes each of dimensions 2 m x 1.25 m x 1 m, then the number of boxes will be .
(a) 1800 (b) 2000 (c) 4000 (d) 8000
Solution.
(c) Given, dimensions of a godown are 40 m, 25 m and 10 m.

$\therefore$ Volume of godown $= 40 \times 25 \times 10$

$= 10000 \text{ m}^3$

Now, volume of each cuboidal box $= 2 \times 1.25 \times 1$

$= 2.5 \text{ m}^3$

$\therefore$ The number of boxes, that can be filled in the godown $= \dfrac{\text{Volume of godown}}{\text{Volume of each cuboidal box}}$

$$= \frac{10000}{2.5} = 4000$$

Question. 11 The volume of a cube is 64 cm^3. Its surface area is
(a) 16 cm^2 (b) 64 cm^2
(c) 96 cm^2 (d) 128 cm^2
Solution.
(c) Let the side of the cube be a. Then,
Volume of cube = a^3 = 64 [given]

⇒ a = 4

Now, surface area of the cube = $a^2 = 6 \times 4^2 = 96\ cm^2$

Question. 12 If the radius of a cylinder is tripled but its curved surface area is unchanged, then its height will be

(a) tripled , (b) constant

(c) one-sixth (d) one-third

Solution.

(d) Let h' be the new height.

Curved surface area of a cylinder with radius r and height h

$$= 2\pi rh$$

Now, according to the question, radius is tripled. Then,

Curved surface area $= 2\pi \times 3r \times h' = 2\pi rh$ [given]

$$\Rightarrow \quad 6\pi r \times h' = 2\pi rh$$

$$\Rightarrow \quad h' = \frac{2\pi rh}{6\pi r}$$

$$\therefore \quad h' = \frac{1}{3}h$$

Hence, the new height will be $\frac{1}{3}$ of the original height.

Question. 13 How many small cubes with edge of 20 cm each can be just accommodated in a cubical box of 2 m edge?

(a) 10 (b) 100 (c) 1000 (d) 10000

Solution.

(c) Volume of cube = $(\text{Side})^3$

Volume of each small cube $= 20^3 = 8000\ cm^3$

$= 0.008\ m^3$

Now, volume of the cubical box $= 2^3 = 8\ m^3$

∴ Number of small cubes, that can just be accommodated in the cubical box

$$= \frac{\text{Volume of cubical box}}{\text{Volume of small cube}} = \frac{8}{0.008}$$

$= 1000$

Question. 14 The volume of a cylinder whose radius r is equal to its height, is

(a) $\frac{1}{4}\pi r^3$ (b) $\frac{\pi r^3}{32}$ (c) πr^3 (d) $\frac{\pi r^3}{8}$

Solution.

(c) Given, r = h

Then, volume of cylinder = $\pi r^2 h = \pi r^2 r = \pi r^3$

Question. 15 The volume of a cube whose edge is 3x, is

(a) $27x^3$ (b) $9x^3$ (c) $6x^3$ (d) $3x^3$

Solution.

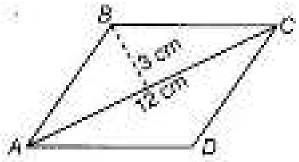
(a) We know that, the volume of a cube = $(\text{Side})^3$

$= a^3$

$= (3x)^3$ [∵ a = 3x, given]

$= 27x^3$

Question. 16 The figure ABCD is a quadrilateral, in which AB area is CD and BC = AD. Its area is



(a) 72 cm^2 (b) 36 cm^2 (c) 24 cm^2 (d) 18 cm^2

Solution.

(b) It is clear from the figure that, quadrilateral ABCD is a parallelogram. The diagonal AC of the given parallelogram ABCD divides it into two triangles of equal areas.

Area of the $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

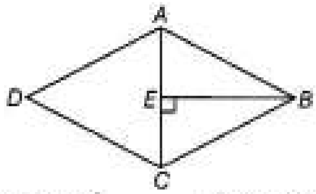
$= \frac{1}{2} \times 12 \times 3 = 18 \text{ cm}^2$

$\therefore$ Area of the parallelogram $ABCD = 2 \times$ Area of ΔABC

$= 2 \times 18$

$= 36 \text{ cm}^2$

Question. 17 What is the area of the rhombus ABCD below, if AC = 6 cm and BE = 4 cm?



(a) 36 cm^2 (b) 16 cm^2 (c) 24 cm^2 (d) 13 cm^2

Solution.

(c) The diagonal AC of the rhombus ABCD divides it into two triangles of equal areas.

Now, area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 6 = 12 \text{ cm}^2$

$\therefore$ Area of the rhombus $ABCD = 2 \times$ Area of ΔABC

$= 2 \times 12 = 24 \text{cm}^2$

Question. 18 The area of a parallelogram is 60 cm^2 and one of its altitude is 5 cm. The length of its corresponding side is

(a) 12 cm (b)6 cm (c) 4 cm (d) 2 cm

Solution.

(a) We know that,

Area of a parallelogram = Side × Altitude

$\Rightarrow \quad a \times h = 60$

$\Rightarrow \quad a \times 5 = 60$

$\Rightarrow \quad a = \frac{60}{5}$

$\therefore \quad a = 12 \text{ cm}$

Question. 19 The perimeter of a trapezium is 52 cm and its each non-parallel side is equal to 10 cm with its height 8 cm. Its area is

(a) 124 cm^2 (b) 118 cm^2 (c) 128 cm^2 (d) 112 cm^2

Solution.

(c) Given, perimeter of a trapezium is 52 cm and each non-parallel side is of 10 cm.

Then, sum of its parallel sides

$= 52 - (10 + 10) = 52 - 20 = 32 \text{ cm}$

$\therefore$ Area of the trapezium $= \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times 32 \times 8 \quad [\because h = 8 \text{ cm and } a + b = 32 \text{ cm}]$

$= 128 \text{ cm}^2$

Question. 20 Area of a quadrilateral ABCD is 20 cm^2 and perpendiculars on BD from opposite vertices are 1 cm and 1.5 cm. The length of BD is

(a) 4 cm (b) 15 cm (c) 16 cm (d) 18 cm

Solution.

(c) Area of the given quadrilateral = $\frac{1}{2}$ (Sum of altitudes) × Corresponding diagonal

$\Rightarrow \quad 20 = \frac{1}{2}(1 + 1.5) \times BD$ [given]

$\Rightarrow \quad \frac{1}{2} \times 2.5 \times BD = 20\ cm^2$

$\Rightarrow \quad BD = 20 \times \frac{2}{2.5} = \frac{40}{2.5} = 16\ cm$

Question. 21 A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted into a cube. The side of the cube is

(a) 6 cm (b) 8 cm (c) 12 cm (d) 24 cm

Solution.

(a) Given, a metal sheet 27 cm long, 8 cm broad and 1 cm thick.

Then, volume of the sheet (cubiodal) = l x b x h

=27 x 8 x 1 = 216 cm^3

Now, since this sheet is melted to form a cube of edge length a (say).

Then, volume of the cube = Volume of the metal sheet

⇒ a^3 =216 cm^3

⇒ a = 6 cm

Hence, the side of the cube is 6 cm.

Question. 22 Three cubes of metal whose edges are 6 cm, 8 cm and 10 cm respectively, are melted to form a single cube. The edge of the new cube is

(a) 12 cm (b) 24 cm

(c) 18 cm (d) 20 cm

Solution.

(a) The edges of three cubes are 6 cm, 8 cm and 10 cm, respectively.

∴ Sum of volumes of the three metal cubes

$= 6^3 + 8^3 + 10^3$ [∵ volume of cube = $(edge)^3$]

$= 216 + 512 + 1000$

$= 1728\ cm^3$

Since, a new cube is formed by melting these three cubes.

Let a be the side of new cube. Then,

Volume of the new cube = Sum of volumes of three metal cubes

$\Rightarrow \quad a^3 = 1728$

$\therefore \quad a = 12\ cm$

Hence, the edge of the new cube is 12 cm.

Question. 23 A covered wooden box has the inner measures as 115 cm, 75 cm and 35 cm and thickness of wood as 2.5 cm. The volume of the wood is

(a) 85000 cm^3 (b) 80000 cm^3

(c) 82125 cm^3 (d) 84000 cm^3

Solution.

(c) Given, inner measures of a wooden box as 115 cm, 75 cm and 35 cm.

Since, thickness of the box is 2.5 cm, then outer measures will be 115 + 5, 75 + 5 and 35 + 5, i.e. 120 cm, 80 cm and 40 cm.

∴ The outer volume = 120 × 80 × 40 = 384000 cm^3

and the inner volume = 115 × 75 × 35 = 301875 cm^3 [∵ volume of cuboid = $l \times b \times h$]

∴ Volume of the wood = Outer volume − Inner volume

$= 384000 - 301875 = 82125\ cm^3$

Question.24 The ratio of radii of two cylinders is 1: 2 and heights are in the ratio 2 : 3. The ratio of their volumes is

(a) 1 : 6 (b) 1 : 9 (c) 1 : 3 (d) 2 : 9

Solution.

(a) Let r_1, r_2 be radii of two cylinders and h_1, h_2 be their heights.

Then, $\frac{r_1}{r_2} = \frac{1}{2}$ and $\frac{h_1}{h_2} = \frac{2}{3}$

Now,
$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{1}{2}\right)^2 \times \frac{2}{3}$$
$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6} = 1:6$$

Hence, $V_1 : V_2 = 1:6$

Question. 25 Two cubes have volumes in the ratio 1 : 64. The ratio of the areas of a face of first cube to that of the other is

(a) 1 : 4 (b) 1 : 8

(c) 1 : 16 (d) 1 : 32

Solution.

(c) Let a and b be the edges of the two cubes, respectively.

Then, according to the question,

$$a^3 : b^3 = 1:64 \quad [\because \text{volume of cube} = (\text{edge})^3]$$
$$\Rightarrow \frac{a^3}{b^3} = \frac{1}{64}$$
$$\Rightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{4}\right)^3$$
$$\Rightarrow \frac{a}{b} = \frac{1}{4} \quad [\text{taking cube roots on both sides}]$$

Now, ratio of areas,
$$\left(\frac{a}{b}\right)^2 = \left(\frac{1}{4}\right)^2 \quad [\because \text{surface area of cube} = 6 \times (\text{edge})^2]$$
$$\Rightarrow \frac{a^2}{b^2} = \frac{1}{16}$$
$$\therefore a^2 : b^2 = 1:16$$

Question. 26 The surface areas of the six faces of a rectangular solid are 16, 16, 32, 32, 72 and 72 sq cm. The volume of the solid (in cu cm) is (a) 192 (b) 384 (c) 480 (d) 2592

Solution.

(a) Since, the solid has rectangular faces.

So, we have
$$l \times b = 16 \quad \ldots(i)$$
$$b \times h = 32 \quad \ldots(ii)$$
$$l \times h = 72 \quad \ldots(iii)$$

where l, b and h are the length, breadth and height respectively, of the solid.

On multiplying Eqs. (i), (ii) and (iii), we get
$$l \times b \times b \times h \times l \times h = 16 \times 32 \times 72$$
$$\Rightarrow l^2 \times b^2 \times h^2 = 36864$$
$$\Rightarrow (lbh)^2 = 36864$$
$$\therefore lbh = 192$$

Hence, the volume of the solid is 192 cu cm.

Question. 27 Ramesh has three containers.

(i) Cylindrical container A having radius r and height h.

(ii)Cylindrical container B having radius 2r and height 1/2 h.

(iii)Cuboidal container C having dimensions r x r x h.

The arrangement of the containers in the increasing order of their volumes is

(a)A,B,C (b) B, C, A

(c) C, A, B (d) Cannot be arranged

Solution.

(c) (i) The volume of the cylindrical container having radius r and height h

$= \pi r^2 h$

(ii) The volume of the cylindrical container with radius 2 r and height 1/2

= π (2 r)^2 x 1/2 h = π x 4 r^2 x 1/2 h

' =2 $\pi r^2 h$

(iii) The volume of the cuboidal container having dimensions rxrxh

= r^2 h

From parts (i), (ii) and (iii), we have the following order C, A, B.

Question. 28 If R is the radius of the base of the hat, then the total outer surface area of the hat is

(a) $\pi r(2h + R)$ (b) $2\pi r(h + R)$ (c) $2\pi rh + \pi R^2$ (d) None of these

Solution.

(c) Given, a cylindrical hat with base radius R and r is radius of the top surface.

Now, total surface area of hat = Curved surface area + Top surface area + Base surface area

$$= 2\pi rh + \pi r^2 + \pi(R^2 - r^2)$$
$$= 2\pi rh + \pi r^2 + \pi R^2 - \pi r^2$$
$$= 2\pi rh + \pi R^2$$

Fill in the Blanks

In questions 29 to 52, fill in the blanks to make the statements are true.

Question. 29 A cube of side 4 cm is painted on all its sides. If it is sliced in 1 cu cm cubes, then number of such cubes that will have exactly two of their faces painted, is_______.

Solution. 24

The volume of a cube of side 4 cm = 4 x 4 x 4 = 64cm^3 When it is sliced into 1 cm^3 cubes, we will get 64 small cubes.

In each side of the larger cube, the smaller cubes in the edges will have more than one face painted.

The cubes which are situated at the corners of the big cube, have three faces painted.

So, to each edge two small cubes are left which have two faces painted. As, the total number of edges in a cube are 12.

Hence, the number of small cubes with two faces painted = 12 x 2 = 24

Question. 30 A cube of side 5 cm is cut into 1 cm cubes. The percentage increase in volume after such cutting is_______.

Solution.

none

Given, a cube of side 5 cm is cut into 1 cm cubes.

The volume of big cube = 5 × 5 × 5

= 125 cm^3

Now, the big cube is cut into 1 cm cubes.

$\therefore$ The number of small cubes = $\dfrac{125}{\text{Volume of 1 small cube}} = \dfrac{125}{1}$

Thus, the volume of big cube = The volume of 125 cubes having an edge 1 cm

Hence, there is no change in the volume.

Question. 31 The surface area of a cuboid formed by joining two cubes of side a face-to-face, is_______.

Solution. $10a^2$
We have, two cubes of side a.
These two cubes are joined face-to-face, then the resultant solid figure is a cuboid which has same breadth and height as the joined cubes has length twice of the length of a cube, i.e. l = 2a,b= aandh = a
Thus, the total surface area of the cuboid = 2 (lb + bh + hi)
= 2 (2a x a + a x a + a x 2a)
= $2(2a^2 + a^2 + 2a^2) = 2 \times 5a^2 = 10a^2$

Question. 32 If the diagonals of a rhombus get doubled, then the area of the rhombus becomes_______its original area.
Solution.
4 times
We know that,
Area of a rhombus $= \frac{1}{2} \times d_1 \times d_2$
where, d_1 and d_2 are diagonals of the rhombus.
If diagonals get doubled, then the area $= \frac{1}{2} 2d_1 \times 2d_2 = 4\left(\frac{1}{2} \times d_1 \times d_2\right)$
Hence, the new area becomes 4 times its original area.

Question. 33 If a cube fits exactly in a cylinder with height h, then the volume of the cube is_______and_______surface area of the cube is .
Solution. h^3, $6h^2$
Since, the cube fits exactly in the cylinder with height h, therefore each side of the cube = h
Now, volume of the cube = $(\text{Side})^3 = h^3$
and surface area of the cube = $6 \times (\text{Side})^2$
$= 6 \times h^2$

Question. 34 The volume of a cylinder becomes_______the original volume, if its radius becomes half of the original radius.
Solution.

$\frac{1}{4}$
The volume of a cylinder with radius r and height $h = \pi r^2 h$
If radius is halved, then new volume $= \pi\left(\frac{r}{2}\right)^2 h = \frac{1}{4}\pi r^2 h$
Hence, the new volume is $\frac{1}{4}$th of the original volume.

Question. 35 The curved surface area of a cylinder is reduced by_______per cent, if the height is half of the original height.
Solution.
50%
The curved surface area of a cylinder with radius r and height $h = 2\pi rh$
If the height is halved, then new curved surface area of cylinder

$$= 2\pi r \frac{h}{2} = \pi rh$$

∴ Percentage reduction in curved surface area $= \frac{2\pi rh - \pi rh}{2\pi rh} \times 100$

$$= \frac{\pi rh}{2\pi rh} \times 100 = 50\%$$

Question. 36 The volume of a cylinder which exactly fits in a cube of side a, is _______.

Solution.

$\frac{\pi a^3}{4}$

Since, the cylinder that exactly fits in cube of side a, has its height equal to the edge of the cube and radius equal to half the edge of the cube.

$\therefore$ Height = a and radius = $\frac{a}{2}$

Now, volume of the cylinder = $\pi r^2 h = \pi\left(\frac{a}{2}\right)^2 a$

$= \frac{1}{4}\pi a^3$

Question. 37 The curved surface area of a cylinder which exactly fits in a cube of side b, is ______.

Solution.

πb^2

Since, the cylinder that exactly fits in a cube of side b, has its height equal to the edge of the cube and radius equal to half the edge of the cube.

$\therefore$ Height = b and radius = $\frac{b}{2}$

Now, curved surface area of the cylinder = $2\pi rh = 2\pi \times \frac{b}{2} \times b$

$= \pi b^2$

Question. 38 If the diagonal d of a quadrilateral is doubled and the heights h1 and h2 falling on d are halved, then the area of quadrilateral is ______.

Solution.

$\frac{1}{2}(h_1 + h_2)d$

Let $ABCD$ be a quadrilateral, where h_1 and h_2 are altitudes on the diagonal $BD = d$.

Then, area of quadrilateral $ABCD = \frac{1}{2}(h_1 + h_2) \times BD$

If altitudes are halved and the diagonal is doubled, then

Area of quadrilateral $ABCD = \frac{1}{2}\left(\frac{h_1}{2} + \frac{h_2}{2}\right) \times 2d = \frac{1}{2}\left(\frac{h_1 + h_2}{2}\right) \times 2d$

$= \frac{1}{2}(h_1 + h_2) \times d$

Question. 39 The perimeter of-a-rectangle becomes______times its original perimeter, if its length and breadth are doubled.

Solution. 2 times

Perimeter of a rectangle with length l and breadth b = 2(l + b)

If the length and the breadth are doubled, then the new perimeter

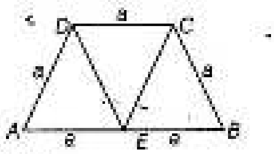
= 2(2l + 2b)

= 2[2(l + b)]

Question. 40 A trapezium with 3 equal sides and one side double the equal side, can be divided into______ equilateral triangles of______area.

Solution.

3, equal



Let $ABCD$ be a trapezium, in which

$AD = DC = BC = a$ (say)

and $AB = 2a$ [given]

Draw medians through the vertices D and C on the side AB.

$\therefore$ $AE = EB = a$

Now, in parallelogram $ADCE$, we have

$AD = EC = a$ and $AE = CD = a$ [$\because$ opposite sides in a parallelogram are equal]

In ΔADE and ΔDEC, $AD = EC$

$AE = CD$

and $DE = DE$ [common]

By SSS $\Delta ADE \cong \Delta DEC$

By triangle rule, $\Delta ADE \cong \Delta DEC$

Thus, ΔADE and ΔDEC are equilateral triangles having equal sides.

Similarly, in parallelogram $DEBC$, we can show that $\Delta DEC \cong \Delta ECB$.

Hence, the trapezium can be divided into 3 equilateral triangles of equal area.

Question. 41 All six faces of a cuboid are_______in shape and of_______area.

Solution. rectangular, different

We know that, a cuboid is made of 6 rectangular plane regions, i.e. 6 rectangular faces, which have different lengths and breadths Therefore the area of the rectangular faces are different. .

Question.42 Opposite faces of a cuboid are_______in area.

Solution. equal

We know that, a cuboid has 6 rectangular races, of which opposite faces have the same length and breadth. Therefore, area of the opposite faces are equal.

Question.43 Curved surface area of a cylinder of radius h and height r is_______.

Solution. $2\pi hr$ (or) $2\pi rh$

We know that, the curved surface area of a cylinder of radius h and height r

= 2π x Radius x Height .

= 2π x h x r=$2\pi hr$

= $2\pi rh$

Question.44 Total surface area of a cylinder of radius h and height r is_______ .

Solution. $2\pi h(r+ h)$

Given, radius of cylinder = h and height of cylinder = r

$\therefore$Total surface area of a cylinder = Curved surface area + Area of top surface + Area of base

= 2 x π x Radius x Height + π (Radius)^2+ π (Radius)^2

= $2\pi hr+ \pi h^2 + \pi h^2$

= $2\pi rh + 2\pi h^2$

= $2\pi h(r + h)$

Question.45 Volume of a cylinder with radiusT? and height r is_______.

Solution. $\pi h^2 r$

Given, radius of cylinder = h and height of cylinder = r.

Now, volume of a cylinder

= π x (Radius)^2 x Height =π x h^2 x r = $\pi h^2 r$

Question.46 Area of a rhombus = 1/2 product of_______.

Solution. diagonals

We know that, the area of a rhombus = Half of the product of its diagonals

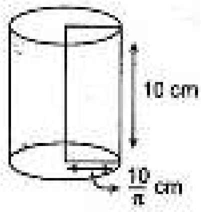
=1/2 [Product of diagonals]

Question. 47 Two cylinders A and B are formed by folding a rectangular sheet of dimensions 20 cm x 10 cm along its length and also along its breadth, respectively. Then, volume of A is_______of volume of B.

Solution.

We have, a rectangular sheet of dimensions $20\,cm \times 10\,cm$.

If we fold it along its length, which is 20 cm, then the resultant figure is a cylinder with height, $h = 10\,cm$ and base circumference, $2\pi r = 20\,cm$



$\Rightarrow \qquad r = \frac{20}{2\pi} = \frac{10}{\pi}\,cm$

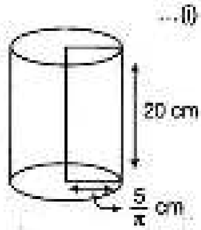
$\therefore$ The volume of the cylinder, so formed $= \pi r^2 h$

$= \pi \times \frac{10}{\pi} \times \frac{10}{\pi} \times 10$

$= \frac{1000}{\pi}\,cm^3$

$= V_1$(say) $\qquad$ ...(i)

Again, if we fold the rectangular sheet along its breadth, which is 10 cm, the figure so obtained is a cylinder with height, $h = 20\,cm$ and the base circumference $2\pi r = 10\,cm$



$\Rightarrow \qquad r = \frac{10}{2\pi} = \frac{5}{\pi}\,cm$

$\therefore$ Volume of the cylinder $= \pi r^2 h = \pi \times \frac{5}{\pi} \times \frac{5}{\pi} \times 20$

$= \frac{500}{\pi}\,cm^3 = V_2$ (say) $\qquad$ ...(ii)

i.e. $\qquad V_2 = 2V_1$

From Eqs. (i) and (ii), we see that the volume of *A* is twice the volume of *B*.

Question. 48 In the above question, curved surface area of A is_______surface area of B.

Solution.

equal

For cylinder *A*, $\qquad h = 10\,cm$ and $r = \frac{10}{\pi}\,cm$

$\therefore$ Curved surface area of $A = 2\pi rh = 2\pi \times \frac{10}{\pi} \times 10 = 200\,cm^2$

Again, for cylinder *B*, $\qquad r = \frac{5}{\pi}\,cm$ and $h = 20\,cm$

$\therefore$ Curved surface area of $B = 2\pi rh = 2\pi \times \frac{5}{\pi} \times 20 = 200\,cm^2$

Hence, the curved surface area of both the cylinders are same.

Question. 49_______of a solid is the measurement of the space occupied by it.

Solution. Volume

We know that, a solid always occupies some space and magnitude of this space region is known as the volume of the solid.

Question. 50_______surface area of room = Area of 4 walls.

Solution. Lateral

We know that, a room is in the shape of a cuboid. Its 4 walls are treated as lateral faces of the cuboid.

$\therefore$ Lateral surface area of room = Area of 4 walls

Question. 51 Two cylinders of equal volume have heights in the ratio 1: 9. The ratio of their radii is_______.

Solution.

3 : 1

Let r_1, r_2 be the radii and h_1, h_2 be the heights of two cylinders.

Given, $\frac{h_1}{h_2} = \frac{1}{9}$

Now, according to the question,

$$\pi r_1^2 h_1 = \pi r_2^2 h_2 \quad [\because \text{volume of cylinder} = \pi r^2 h]$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{h_2}{h_1}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{9}{1}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{9}}{1}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{3}{1}$$

Hence, $r_1 : r_2 = 3 : 1$

Question. 52 Two cylinders of same volume have their radii in the ratio 1 : 6. Then, ratio of their heights is______.

Solution.

36 : 1

Let r_1, r_2 be the radii and h_1, h_2 be the heights of two cylinders.

Given, $\frac{r_1}{r_2} = \frac{1}{6}$

Now, according to the question,

$$\pi r_1^2 h_1 = \pi r_2^2 h_2 \quad [\because \text{volume of cylinder} = \pi r^2 h]$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{h_2}{h_1}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{h_2}{h_1}$$

$$\Rightarrow \left(\frac{1}{6}\right)^2 = \frac{h_2}{h_1}$$

$$\Rightarrow \frac{1}{36} = \frac{h_2}{h_1}$$

$$\text{or} \quad \frac{h_1}{h_2} = \frac{36}{1}$$

$$\text{or} \quad h_1 : h_2 = 36 : 1$$

True/False

In questions 53 to 61, state whether the statements are True or False.

Question. 53 The areas of any two faces of a cube are equal.

Solution. True

Since, all the faces of a cube are squares of same side length, therefore the areas of any two faces of a cube are equal.

Question. 54 The areas of any two faces of a cuboid are equefl.

Solution. False

A cuboid has rectangular faces with different lengths and breadths. Only opposite faces of cuboid have the same length and breadth.

Therefore, areas of only opposite faces of a cuboid are equal.

Question. 55 The surface area of a cuboid formed by joining face-to-face 3 cubes of side x is 3 times the surface area of a cube of side x.

Solution. False

Three cubes having side x are joined face-to-face, then the cuboid so formed has the same

height and breadth as the cubes but its length will be thrice that of the cubes.
Hence, the length, breadth and height of the cuboid so formed are 3x, x and x, respectively.
Then, its surface area = 2 (lb + bh + hl)
$= 2(3x \times x + x \times x + x \times 3x) = 2(3x^2 + x^2 + 3x^2)$
$= 2 \times 7x^2 = 14x^2$
Now, the surface area of the cube of side $x = 6\ (\text{Side})^2 = 6x^2$ Hence, the statement is false.

Question. 56 Two cuboids with equal volume will always have equal surface area.
Solution.
False
We discard the statement by a counter example.
Let the dimensions of two cuboids be 1 cm × 1 cm × 2 cm and 1 cm × $\frac{1}{2}$ cm × 4 cm, respectively.
Then, volume of first cuboid $= l \times b \times h = 1 \times 1 \times 2 = 2\ cm^3$
and volume of second cuboid $= l \times b \times h = 1 \times \frac{1}{2} \times 4 = 2\ cm^3$
Now, the surface area of first cuboid $= 2\ (lb + bh + hl)$
$= 2\ (1 \times 1 + 1 \times 2 + 2 \times 1)$
$= 2\ (1 + 2 + 2) = 10\ cm^2$
and surface area of the second cuboid $= 2(lb + bh + hl)$
$= 2\left(1 \times \frac{1}{2} + \frac{1}{2} \times 4 + 1 \times 4\right) = 2\left(\frac{1}{2} + \frac{4}{2} + 4\right)$
$= 2\left(\frac{5}{2} + 4\right) = 2\left(\frac{13}{2}\right) = 13\ cm^2$
which are not equal.
So, the statement is false.

Question. 57 The area of a trapezium becomes 4 times, if its height gets doubled.
Solution. False
We know that,
Area of a trapezium = 1/2 (a + b) x h
where, a and b are the lengths of parallel sides and h is the altitude (height).
Now, if the height gets doubled, then
Area of trapezium = 1/2(a + b) x 2h = 2(1/2(a + b) x h)
Hence, the area is doubled.
So, the statement is false.

Question. 58 A cube of side 3 cm painted on all its faces, when sliced into 1 cu cm cubes, will have exactly 1 cube with none of its faces painted.
Solution. True
Given, a cube of side 3 cm is painted on all its faces. Now, it is sliced into 1 cu cm cubes.
Then, there will be 8 corner cubes that have 3 sides painted, 6 centre cubes with only one side painted and only 1 cube in the middle that has no side painted.

Question. 59 Two cylinders with equal volume will always have equal surface area.
Solution. False

Consider two cylinders with the following measures

e.g. $r = 2$ cm, $h = 9$ cm and $r = 3$ cm, $h = 4$ cm

For the first cylinder,

$$\text{Volume} = \pi r^2 h = \pi \times 2^2 \times 9 = 36\pi \text{ cm}^3$$

Again, for the second cylinder,

$$\text{Volume} = \pi r^2 h = \pi \times 3^2 \times 4$$
$$= 36\pi \text{ cm}^3$$

∴ The volumes are equal.

Now, surface area of first cylinder $= 2\pi rh = 2\pi \times 2 \times 9 = 36\pi \text{ cm}^2$

and surface area of second cylinder $= 2\pi rh = 2\pi \times 3 \times 4 = 24\pi \text{ cm}^2$

which are not equal.

So, the statement is false.

Question. 60 The surface area of a cube formed by cutting a cuboid of dimensions 2 x 1 x 1 in 2 equal parts, is 2 sq units.

Solution. False

The dimensions of the given cuboid are 2 x 1 x 1. It is sliced into two equal parts, which are cubes.

Then, the dimensions of the cube, so formed are 1 x 1 x 1.

∴The surface area of the cube so formed = 6 (Side)^2 = 6 x (1)^2 = 6sq units

Hence, the surface area of the sliced cube is 6 sq units.

Question.61 Ratio of area of a circle to the area of a square whose side equals radius of circle, is 1 : TC.

Solution. False

Given, side of a square equals radius of a circle.

Then, area of the square = r^2

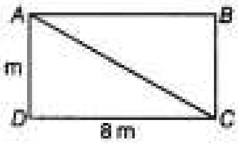
and area of the circle =πr^2

where r is a radius of the circle.

Now, the ratio of area of the circle to area of the square = $\pi r^2 : r^2 = \pi : 1$.

Question. 62 The area of a rectangular field is 48 m^2 and one of its sides is 6m. How long will a lady take to cross the field diagonally at the rate of 20 m/min?

Solution.



Given, the area of a rectangular field is 48 m^2 and one side of the rectangle is = 6 m.

∴ Area of a rectangle = Length × Breadth

⇒ 48 = 6 × Breadth

⇒ Breadth = 8 m.

In ΔACD, $\angle D = 90°$ [∵ in a rectangle, all angles are of 90°]

So, it is a right-angled triangle.

By using Pythagoras theorem, we have

$$(AC)^2 = (AD)^2 + (DC)^2$$
$$\Rightarrow (AC)^2 = (6)^2 + (8)^2$$
$$\Rightarrow (AC)^2 = 36 + 64$$
$$\Rightarrow AC = \sqrt{100}$$
$$\therefore AC = 10 \text{ m}$$

Time taken by lady to cross the field diagonally at rate of 20 m/min $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{10}{20} = \frac{1}{2} \text{ min or } 30 \text{ s}$$

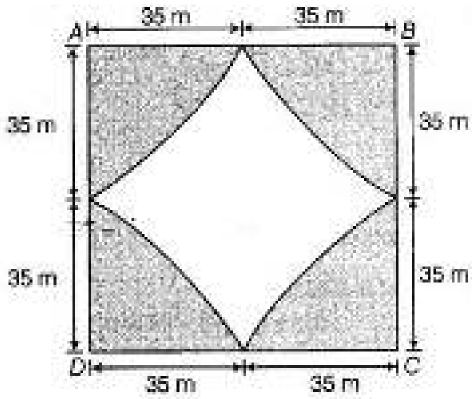
Question. 63 The circumference of the front wheel of a cart is 3 m long and that of the back wheel is 4 m long. What is the distance travelled by the cart, when the front wheel makes five more revolutions than the rear wheel?

Solution.

Given, circumference of front wheel = 3 m

Now, distance covered by front wheel of the cart in 1 revolution
= Circumference of front wheel .
∴ Distance covered by front wheel in 5 revolutions = 3 x 5 = 15 m
Hence, the distance covered by the cart is 15 m.

Question. 64 Four horses are tethered with equal ropes at 4 corners of a square field of side 70 m, so that they just can reach one-another. Find the area left ungrazed by the horses.
solution.
Given, side of a square = 70 m
Also, four horses are tethered with equal ropes at 4 corners of the square field. Hence, each horse can graze upto 35 m of distance along the side.



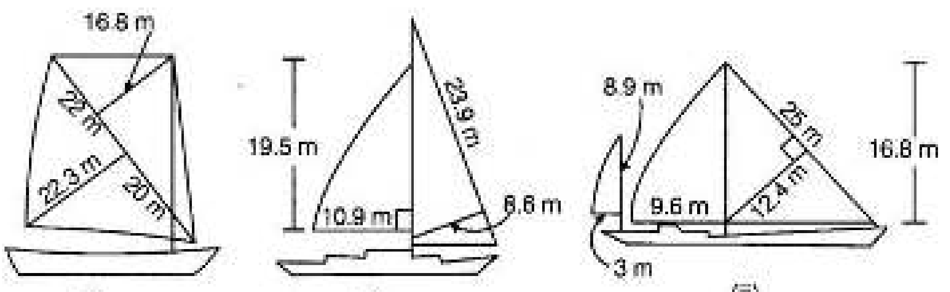
∴ Area of the square field = Side × Side
$= 70 \times 70$
$= 4900 \text{ m}^2$

The grazed area is making a complete circle by taking all the four grazed parts.
So, area of grazed part $= \pi r^2$
$= \frac{22}{7} \times 35 \times 35$ [∵ $r = 35$ m]
$= 22 \times 5 \times 35$
$= 3850 \text{ m}^2$

Area left ungrazed by the horses = Area of square field − Area of grazed part
$= 4900 - 3850$
$= 1050 \text{ m}^2$

Question. 65 The walls and ceiling of a room are to be plastered. The length, breadth and height of the room are 4.5 m, 3m and 350 cm, respectively. Find the cost of plastering at the rate of ? 8 per m2.
solution.
Given, length of the room $(l) = 4.5$ m
Breadth of the room $(b) = 3$ m
Height of the room $(h) = 350 \text{ cm} = 3.5$ m [∵ 100 cm = 1 m]
and the cost of plastering = ₹ 8 per m^2
∴ Area of the walls $= 2h(l + b)$
$= 2 \times 3.5 (4.5 + 3)$
$= 7 \times (7.5) = 52.5 \text{ m}^2$
Area of the ceiling $= lb = 4.5 \times 3 = 13.5 \text{ m}^2$
Area of the room $= 52.5 + 13.5 = 66 \text{ m}^2$
Hence, the cost of plastering $= 66 \times 8 =$ ₹ 528

Question. 66 Most of the sailboats have two sails, the jib and the mainsail. Assume that the sails are triangles. Find the total area of each sail of the sailboats to the nearest tenth.



(i) (ii) (iii)

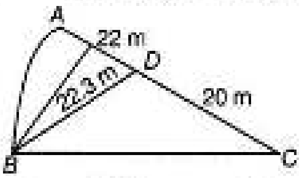
solution.

In the sailboat (i),

Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

In ΔABC, AC = Base = 22 + 20 = 42 m

BD = Height = 22.3 m



$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 42 \times 22.3 = \frac{936.6}{2} = 468.3 \text{ m}^2$

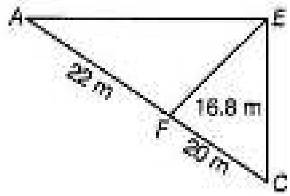
In another triangular part,

In ΔACE EF = Height = 16.8 m

AC = Base = 22 + 20 = 42 m

$\therefore$ Area of $\Delta ACE = \frac{1}{2} \times 42 \times 16.8$

$= \frac{705.6}{2} = 352.8 \text{ m}^2$



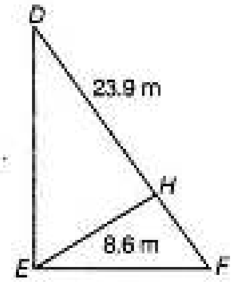
$\therefore$ Area of sailboat (i) = 468.3 + 352.8 = 821.1 m^2

In sailboat (ii),

Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

In ΔABC, $\angle B = 90°$, base (BC) = 10.9 m and height (AB) = 19.5 m

Area of $\Delta ABC = \frac{1}{2} \times 10.9 \times 19.5 = \frac{212.55}{2} = 106.275 \text{ m}^2$



In another triangular part,

Area of $\Delta DEF = \frac{1}{2} \times DF \times EH$

$= \frac{1}{2} \times 23.9 \times 8.6$

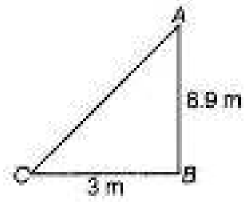
$= \frac{205.54}{2} = 102.77 \text{ m}^2$

Area of sailboat (ii) = 106.275 + 102.77 = 209.045 m^2

In sailboat (iii),

Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

In ΔABC, $AB = 8.9$ m and $BC = 3$ m.

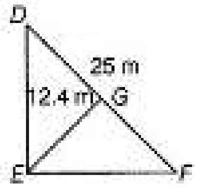


Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$$= \frac{1}{2} \times 8.9 \times 3$$

$$= \frac{26.7}{2} = 13.35 \text{ m}^2$$

In another triangular part,

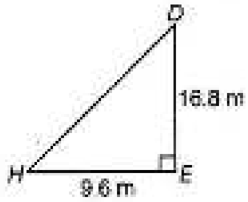


Area of $\Delta DEF = \frac{1}{2} \times DF \times EG$

$$= \frac{1}{2} \times 25 \times 12.4$$

$$= 155 \text{ m}^2$$

In another triangular part,



Area of $\Delta DEH = \frac{1}{2} \times DE \times EH$

$$= \frac{1}{2} \times 9.6 \times 16.8$$

$$= 80.64 \text{ m}^2$$

$\therefore$ Area of sailboat (iii) $= 155 + 80.64$

$$= 235.64 \text{ m}^2$$

Question. 67 The area of a trapezium with equal non-parallel sides is 168 m^2. If the lengths of the parallel sides are 36 m and 20 m, then find the length of the non-parallel sides.

solution.

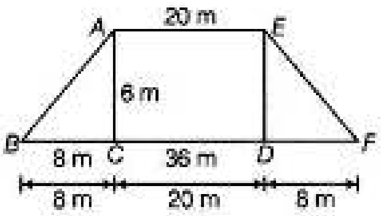
Length of the parallel sides are 36 m and 20 m.

Area of a trapezium = 168 m^2

$\because$ Area of a trapezium $= \frac{1}{2} \times$ [Sum of parallel sides] $\times$ Height

$\therefore \quad 168 = \frac{1}{2} \times [36 + 20] \times \text{Height}$

$\Rightarrow \quad \text{Height} = \frac{168 \times 2}{56} = \frac{336}{56} = 6 \text{ m}$



In ΔACB, using Pythagoras theorem,

$$(AB)^2 = (BC)^2 + (AC)^2$$

$\Rightarrow \quad (AB)^2 = (8)^2 + (6)^2$

$\Rightarrow \quad (AB)^2 = 64 + 36$

$\Rightarrow \quad (AB)^2 = 100$

$\therefore \quad AB = \sqrt{100} = 10 \text{ m}$

Hence, length of the non-parallel side is 10 m.

Question. 68 Mukesh walks around a circular track of radius 14 m with a speed of 4 km/h. If he takes 20 rounds of the track, for how lontj does he walk?

solution.

Radius of the circular track = 14 m

Circumference of the circular track $= 2\pi r = 2 \times \frac{22}{7} \times 14$

$= 44 \times 2 = 88$ m

Total distance cover in 20 rounds = 88 × 20 = 1760 m

Speed of Mukesh on the circular track = 4 km/h

$= \frac{4 \times 1000}{60} = \frac{2 \times 100}{3} = \frac{200}{3}$ m/min

Time taken by Mukesh $= \frac{1760}{200/3} = \frac{1760 \times 3}{200} = \frac{176 \times 3}{20} = 26.4$ min

26.4 min = 26 min and 24 s

Question. 69 The areas of two circles are in the ratio 49 : 64. Find the ratio of their circumferences.

solution.

Given, the area of two circles are in the ratio 49 : 64.

Area of a circle $= \pi r^2$

Let area of the first circle $= \pi r_1^2$

and area of the second circle $= \pi r_2^2$

According to the question, $\frac{49}{64} = \frac{\pi r_1^2}{\pi r_2^2}$

$\Rightarrow \frac{49}{64} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \frac{(7)^2}{(8)^2} = \frac{r_1^2}{r_2^2} \Rightarrow \left(\frac{7}{8}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$

$\therefore r_1 = 7$ and $r_2 = 8$

The ratio of circumferences of these two circles

$= \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{7}{8}$ [$\because$ circumference of circle $= 2\pi r$]

Hence, required ratio is 7 : 8.

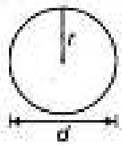
Question. 70 There is a circular pond and a footpath runs along its boundary. A person walks around it, exactly once keeping close to the edge. If his step is 66 cm long and he takes exactly 400 steps to go around the pond, then find the diameter of the pond.

solution.

Let the radius of the pond be r. Then, diameter of the pond, $d = 2 \times r$

[$\because$ diameter = 2 × radius]

Since, a person takes exactly 400 steps with 66 cm long each step to go round the pond.



Hence, the circumference of the pond = 66 × 400

= 26400 cm

$= \frac{26400}{100}$ m = 264 m [$\because$ 100 cm = 1 m]

We know that, circumference of a circle is $2\pi r$.

$\therefore 2\pi r = 264$

$\Rightarrow r = \frac{264}{2} \times \frac{7}{22} = \frac{264 \times 7}{44} = 42$ m

Hence, radius of the pond = 42 m

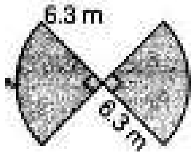
So, diameter of the pond = 2 × 42 = 84 m

Question. 71 A running track has 2 semi-circular ends of radius 63 m and two straight lengths. The perimeter of the track is 1000 m. Find each straight length.

solution.

Radius of semi-circular track = 63 m
Perimeter of 2 semi-circles = Perimeter of 1 circle
Perimeter of a circular track = $2\pi r$
Perimeter of circular track $= 2 \times \frac{22}{7} \times 63$
$= 2 \times 22 \times 9$
$= 44 \times 9$
$= 396$ m
$\because$ The perimeter of the total track is 1000 m.
$\therefore$ Length of two straight lengths track
$= 1000 - 396$
$= 604$ m
Length of 1 straight length track $= \frac{604}{2} = 302$ m

Question. 72 Find the perimeter of the given figure.



solution.
Radius of the given figure = 6.3 m
Two sections in figure form a semi-circle.
Perimeter of semi-circular figure $= \frac{2\pi r}{2} + 2r = \pi r + 2r$
$= \frac{22}{7} \times 6.3 + 2 \times 6.3$
$= 22 \times 0.9 + 2 \times 6.3$
$= 19.8 + 12.6 = 32.4$ m

Question. 73 A bicycle wheel makes 500 revolutions in moving 1 km. Find the diameter of the wheel.
solution.
A bicycle wheel makes 500 revolutions in moving 1 km.
In 1 revolution, the bicycle wheel covers
$= \frac{1}{500}$ km $= \frac{1000}{500}$ m $= 2$ m [$\because$ 1 km = 1000 m]
1 revolution distance = Circumference/Perimeter of the wheel
$\Rightarrow$ $2\pi r = 2$
$\Rightarrow$ $2 \times \frac{22}{7} \times r = 2$
$\Rightarrow$ $r = \frac{2 \times 7}{2 \times 22} = \frac{7}{22}$ [$\because$ circumference of a circle $= 2\pi r$]
$\therefore$ Diameter (d) $= 2r = \frac{7}{22} \times 2 = \frac{7}{11} = 0.636$ m

Question. 74 A boy is cycling such that the wheels of the cycle are making 140 revolutions per hour. If the diameter of the wheel is 60 cm, then calculate the speed (in km/h) with which the boy is cycling.
solution.

The cycle makes 140 revolutions per hour.

Diameter of the wheel = 60 cm

Radius of the wheel = 30 cm

Circumference of a circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 30$$

$$= \frac{44 \times 30}{7}$$

$$= 188.57 \text{ cm}$$

Distance cover in 140 revolutions

$$= 140 \times 188.57$$

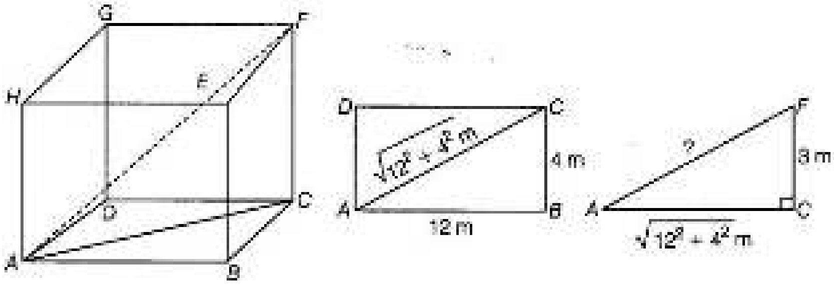
$$= 26400 \text{ cm}$$

$$\therefore \quad \text{Speed} = \frac{26400}{100000} \text{ km/h}$$

$$= 0.264 \text{ km/h} \qquad [\because 1 \text{ km} = 100000 \text{ cm}]$$

Question. 75 Find the length of the largest pole, that can be placed in a room of dimensions 12 mx 4 m x 43 m.

solution.



We have, ΔACF, in which $\angle C = 90°$, $CF = 3$ m and $AC = \sqrt{(12)^2 + (4)^2}$ m

The length of the largest pole = Length of diagonal of cuboid (in shape of room)

$$\Rightarrow \quad (AF)^2 = (AC)^2 + (CF)^2$$

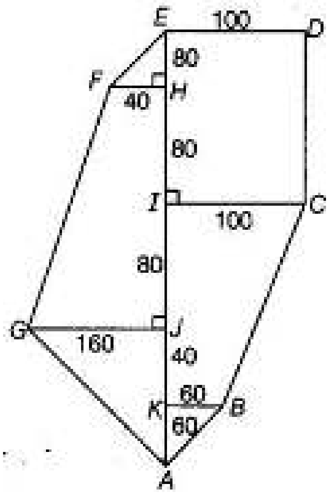
$$\Rightarrow \quad (AF)^2 = (12)^2 + (4)^2 + (3)^2$$

$$\Rightarrow \quad AF = \sqrt{144 + 16 + 9}$$

$$= \sqrt{169}$$

$$= 13 \text{ m}$$

Question. 76 Find the area of the following fields. All dimensions are in metres.



solution.

Area of the given figure

= Area of ΔEFH + Area of rectangle $EDCI$ + Area of trapezium $FHJG$ + Area of trapezium $ICBK$ + Area of Δ GJA + Area of ΔKBA

Area of $\Delta EFH = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 40 \times 80 = 40 \times 40 = 1600 \text{ m}^2$

Area of rectangle $EDCI$ = Length × Breadth = $100 \times 160 = 16000 \text{ m}^2$

Area of trapezium $FHJG = \frac{1}{2} \times$ [Sum of parallel sides] × Height

$= \frac{1}{2} \times [40 + 160] \times 160$

$= \frac{200}{2} \times 160 = 100 \times 160$

$= 16000 \text{ m}^2$

Area of trapezium $ICBK = \frac{1}{2} \times$ [Sum of parallel sides] × Height

$= \frac{1}{2} \times [60 + 100] \times 120$

$= \frac{1}{2} \times 160 \times 120$

$= 80 \times 120 = 9600 \text{ m}^2$

Area of $\Delta AJG = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 160 \times 100$

$= 80 \times 100 = 8000 \text{ m}^2$

Area of $\Delta KBA = \frac{1}{2} \times \text{Base} \times \text{Height}$

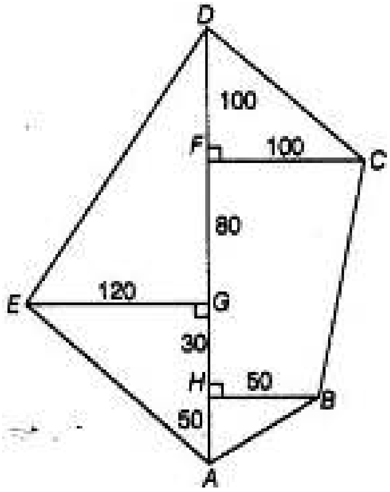
$= \frac{1}{2} \times 60 \times 60 = 60 \times 30 = 1800 \text{ m}^2$

Thus, the area of the complete figure

$= 1600 + 16000 + 16000 + 9600 + 8000 + 1800$

$= 53000 \text{ m}^2$

Question. 77 Find the area of the following fields. All dimensions are in metres.



solution.

Area of the given figure = Area of ΔDCF + Area of ΔEGD + Area of trapezium $FCBH$ + Area of ΔEGA + Area of ΔAHB

$\therefore$ Area of $\Delta DCF = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 100 \times 100$

$= \frac{10000}{2} = 5000\, m^2$

Area of $\Delta EGD = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 120 \times 180$

$= 60 \times 180 = 10800\, m^2$

Area of trapezium $= \frac{1}{2} \times [\text{Sum of parallel sides}] \times \text{Height}$

$= \frac{1}{2} \times [100 + 50] \times 110 = \frac{1}{2} \times 150 \times 110$

$= 75 \times 110 = 8250\, m^2$

Area of $\Delta EGA = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 120 \times 80$

$= 60 \times 80 = 4800\, m^2$

Area of $\Delta AHB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 50 \times 50$

$= 25 \times 50$

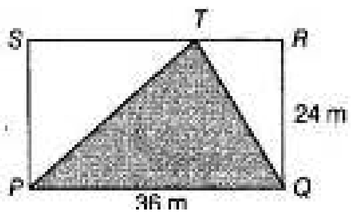
$= 1250\, m^2$

Thus, the area of the complete figure $= 5000 + 10800 + 8250 + 4800 + 1250$

$= 30100\, m^2$

In questions from 78 to 85, find the area of the shaded portion in the following figures.

Question. 78



solution.

Area of the shaded portion = Area of ΔPTQ

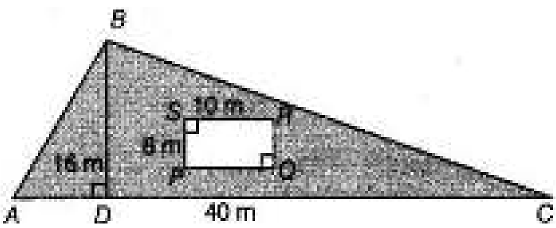
$\because$ Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

So, in ΔPTQ, RQ = Height

$\therefore$ Area of $\Delta PTQ = \frac{1}{2} \times 36 \times 24$

$= 18 \times 24 = 432\, m^2$

Question. 79



solution.

Area of shaded region = Area of ΔABC – Area of rectangle $PQRS$

$\therefore$ Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 40 \times 16\ m^2$

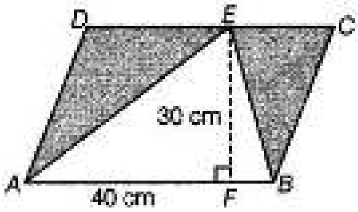
$= 20 \times 16 = 320\ m^2$

$\because$ Area of a rectangle = Length $\times$ Breadth

$\therefore$ Area of the rectangle $= 10 \times 8 = 80\ m^2$

Area of shaded region $= 320 - 80 = 240\ m^2$

Question.80



solution.

Area of shaded region = Area of the parallelogram $ABCD$ – Area of ΔABE

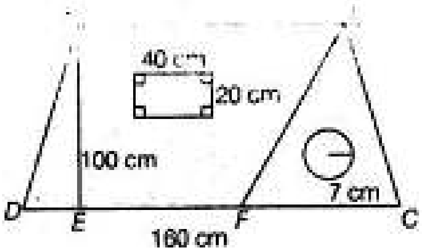
$\because$ Area of a parallelogram = Side $\times$ Height

$\therefore$ Area of a parallelogram $ABCD = 40 \times 30 = 1200\ cm^2$

Area of $\Delta AEB = \frac{1}{2} \times AB \times EF = \frac{1}{2} \times 40 \times 30 = 600\ cm^2$

$\therefore$ Area of shaded region $= 1200 - 600 = 600\ cm^2$

Question.81



solution.

Area of shaded portion = Area of trapezium – Area of rectangle – Area of circle

Area of trapezium $= \frac{1}{2} \times$ [Sum of parallel sides] $\times$ Height

$= \frac{1}{2} \times (120 + 160) \times 100$

$= \frac{1}{2} \times 280 \times 100$

$= \frac{28000}{2} = 14000\ cm^2$

Area of rectangle = Length $\times$ Breadth $= 40 \times 20 = 800\ cm^2$

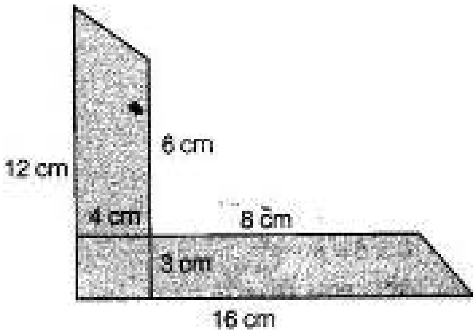
Area of circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7$

$= 154\ cm^2$

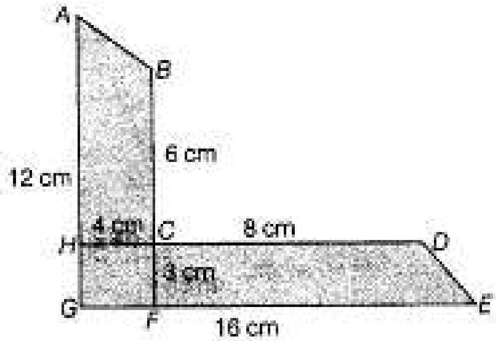
$\therefore$ Area of shaded portion $= 14000 - 800 - 154$

$= 13046\ cm^2$

Question.82



solution.



Area of shaded portion = Area of trapezium *ABCH* + Area of trapezium *CDEF*

Area of trapezium = $\frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Height)

$\therefore$ Area of trapezium *ABCH*

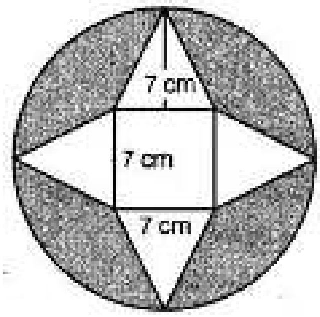
$$= \frac{1}{2} \times (12 + 6) \times 4 = \frac{1}{2} \times 18 \times 4 = 18 \times 2 = 36 \text{ cm}^2$$

$\therefore$ Area of trapezium *CDEF* $= \frac{1}{2} \times (8 + 16) \times 3$

$$= \frac{24 \times 3}{2} = 36 \text{ cm}^2$$

Area of shaded portion = 36 + 36 = 72 cm^2

Question.83



solution.

Area of shaded region = Area of the circle – Area of four triangles – Area of a square

Area of four triangles = $4 \times \frac{1}{2} \times$ Base $\times$ Height

$$= 4 \times \frac{1}{2} \times 7 \times 7 = \frac{4 \times 49}{2} = 2 \times 49 = 98 \text{ cm}^2$$

Area of a square = $(\text{Side})^2 = (7)^2 = 49 \text{ cm}^2$

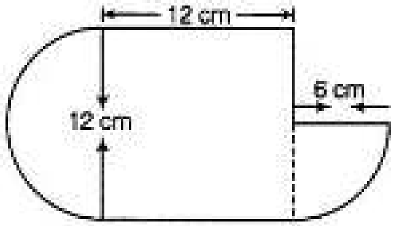
Area of the circle = $\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{11 \times 3 \times 21}{2} = \frac{693}{2}$

$$= 346.5 \text{ cm}^2$$

Area of shaded region = (346.5 – 98 – 49)

$$= 199.5 \text{ cm}^2$$

Question.84



solution.

Area of the given figure = Area of section with 6 cm + Area of square with side measure 12 cm + Area of semi-circle with radius 6 cm

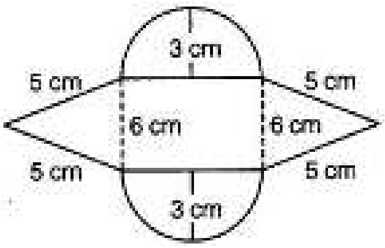
$$= \frac{22 \times 6 \times 6}{7 \times 4} + (12)^2 + \frac{22 \times 6 \times 6}{7 \times 2} = \frac{11 \times 18}{7} + 144 + \frac{11 \times 36}{7}$$

$$= \frac{198}{7} + 144 + \frac{396}{7}$$

$$= \frac{198 + 1008 + 396}{7} = \frac{1602}{7} = 228.85 \text{ cm}^2$$

Hence, area of the given figure is 225.85 cm^2.

Question.85



solution.

Area of the given figure = Area of two semi-circles + Area of two triangles + Area of a square

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [$\because a = 5$ cm, $b = 5$ cm and $c = 6$ cm, given]

where, $s = \frac{a+b+c}{2} = \frac{5+5+6}{2}$

$= \frac{16}{2} = 8$ cm

$\therefore$ Area of a triangle $= \sqrt{8(8-5)(8-5)(8-6)}$

$= \sqrt{8 \times 3 \times 3 \times 2} = \sqrt{144} = 12 \text{ cm}^2$

$\therefore$ Area of 2 triangles $= 2 \times 12 = 24 \text{ cm}^2$

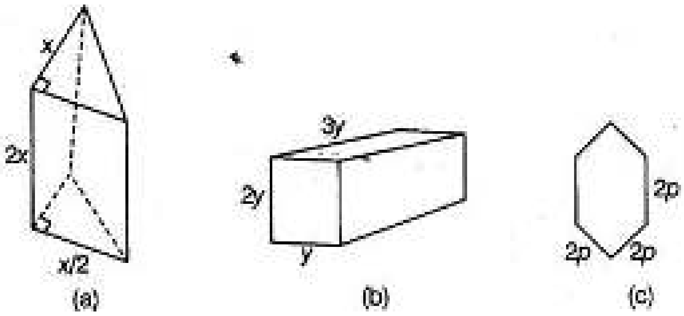
$\therefore$ Area of 2 semi-circles = Area of 1 circle

Area of a circle $= \pi r^2 = \frac{22}{7} \times 3 \times 3 = \frac{9 \times 22}{7} = \frac{198}{7} = 28.28 \text{ cm}^2$

$\therefore$ Area of a square $= 6 \times 6 = 36 \text{ cm}^2$

Area of the given figure $= (24 + 28.28 + 36) = 88.28 \text{ cm}^2$

Question.86 Find the volume of each of the given figure, if Volume = Base area x Height



solution.

∵ Volume of each of the given figure = Base area × Height

In Fig. (a), base is rectangle.

So, area of rectangle $= 2x \times \frac{x}{2} = x^2$

∴ Height $= \frac{x}{2}$

∴ Volume of the figure $= x^2 \times \frac{x}{2} = \frac{x^3}{2}$

In Fig. (b), base is rectangle.

So, area of rectangle $= y \times 3y = 3y^2$

Height $= 2y$

∴ Volume of the figure $= 3y^2 \times 2y = 6y^3$

In Fig. (c), base is rectangle.

So, area of rectangle $= 2p \times 2p = 4p^2$

Height $= 2p$

Volume of the figure $= 4p^2 \times 2p = 8p^3$

Question.87 A cube of side 5 cm is cut into as many 1 cm cubes as possible. What is the ratio of the surface areas of the original cube to that of the sum of the surface areas of the smaller cubes?

solution.

Surface area of a cube = 6 a^2, where a is side of a cube.

∴ Side of cube = 5cm

∴ Surface area of the cube = 6 x $(5)^2$ = 6 x 25

= 150cm^2

Now, surface area of the cube with side 1 cm = 6 x $(1)^2$= 6 cm^2

∴ Surface area of 5 cubes with side 1 cm = 5 x 6 = 30 cm^2

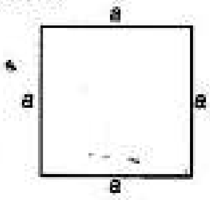
Ratio of the surface area of the original cube to that of the sum of the surface area of the smaller cubes

=30/150=3/15=1:5

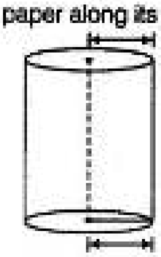
Question. 88 A square sheet of paper is converted into a cylinder by rolling it along its side. What is the ratio of the base radius to the side of the square?

solution.

Let the sides of a square paper be a.



A cylinder is formed by rolling the paper along its side.



∴ Base of the cylinder is circle, so the circumference of the circle is equal to the length of each side of the square sheet.

⇒ $2\pi r = a$ [∵ circumference of circle = $2\pi r$]

∴ $r = \frac{a}{2\pi}$

∴ Ratio $= \frac{a}{2\pi} : a = \frac{1}{2\pi} : 1 = 1 : 2\pi$

Hence, the ratio of the base radius to the side of the square is $1 : 2\pi$.

Question. 89 How many cubic metres of Earth must be dug to construct a well 7 m deep and of diameter 2.8 m?
solution.

A well is in the form of cylinderical form.
Earth must be dug to construct a well 7 m deep and diameter 2.8 m is equal to the volume of a cylinder with 7 m height and diameter 2.8 m.
Volume of a cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{2.8}{2} \times \frac{2.8}{2} \times 7 = 11 \times 2.8 \times 1.4 = 43.12 \text{ m}^3$$

Question. 90 The radius and height of a cylinder are in the ratio 3 : 2 and its volume is 19404 cm3. Find its radius and height.
solution.

The radius and height of a cylinder are in the ratio 3 : 2.
Let the radius be $3x$ and height be $2x$. Then,
Volume of cylinder = 19404 cm^3 [given]
$\because$ Volume of a cylinder = $\pi r^2 h$

$\therefore$ $$19404 = \frac{22}{7} \times (3x)^2 \times (2x)$$

$\Rightarrow$ $$19404 = \frac{22}{7} \times 9x^2 \times 2x = \frac{22 \times 18x^3}{7}$$

$\Rightarrow$ $$19404 = \frac{396x^3}{7}$$

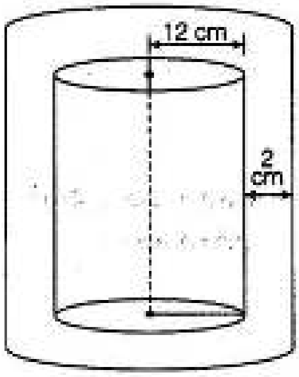
$\Rightarrow$ $$x^3 = 343 \Rightarrow x^3 = (7)^3$$

$\therefore$ $$x = 7 \text{ cm}$$

Hence, radius of the cylinder = $3 \times 7 = 21$ cm
and height of the cylinder = $2 \times 7 = 14$ cm

Question. 91 The thickness of a hollow metallic cylinder is 2 cm. It is 70 cm long with outer radius of 14 cm. Find the volume of the metal used in making the cylinder, assuming that it is open at both the ends. Also, find its weight if the metal weighs 8 g per cm^3.
solution.

The thickness of the hollow metallic cylinder is 2 cm.
Height of the cylinder = 70 cm
Outer radius of the cylinder = 14 cm
Inner radius of the cylinder = 14 – 2 = 12 cm



Volume of the metal used in making the cylinder = Volume of the hollow cylinder

$$= \pi(R^2 - r^2) \times h$$

$$= \frac{22}{7} \times [(14)^2 - (12)^2] \times 70$$

$$= 22 \times [196 - 144] \times 10$$

$$= 22 \times 52 \times 10 = 11440 \text{ cm}^3$$

Weight of 11440 cm^3, if metal is 8 g per cm^3

$$= 11440 \times 8 = 91520 \text{ g}$$

Question. 92 Radius of a cylinder is r and the height is h. Find the change in the volume if the
(i) height is doubled.
(ii)height is doubled and the radius is halved.
(iii)height remains
solution.

$\because$ Volume of a cylinder $= \pi r^2 h$

where, h is height and r is radius of base of the cylinder.

(i) If height is double i.e. $h = 2 \times h = 2h$

Then, its volume $= \pi r^2 \times 2h$

$= 2\pi r^2 h$

Hence, volume became double of original volume.

(ii) If height is doubled and the radius is halved,

i.e. $h = 2h$ and $r = \frac{r}{2}$

$\therefore$ Volume $= \pi \times \left(\frac{r}{2}\right) \times \left(\frac{r}{2}\right) \times 2h$

$= \pi \times \frac{r^2}{4} \times 2h = \frac{\pi r^2 h}{2}$

Hence, volume became half of the original volume.

(iii) If height remains same and radius is halved,

i.e. $h = h$ and $r = \frac{r}{2}$

$\therefore$ Volume $= \pi \times \frac{r}{2} \times \frac{r}{2} \times h = \pi \times \frac{r^2}{4} \times h$

Hence, volume became $\frac{1}{4}$ th of the original volume.

Question. 93 If the length of each edge of a cube is tripled, what will be the change in its volume?
solution.

Let the edge of a cube be a.

If edge of the cube became tripled i.e. a = 3 x a = 3a

$\because$ Volume of the cube = a^3

$\therefore$ Volume of the cube with edge tripled = $(3a)^3 = 27\ a^3$

Hence, volume is 27 times of the original volume.

Question. 94 A carpenter makes a box which has a volume of 13400 cm^3 . The base has an area of 670 cm^2 . What is the height of the box?
solution.

Let the height of the box be h.

Volume of the box $= 13400\ cm^2$

Area of base of the box $= 670\ cm^2$

$\because$ Volume of a box = Area of base × Height

$\therefore \quad 13400 = 670 \times h$

$\Rightarrow \quad h = \frac{13400}{670}$

$\Rightarrow \quad h = \frac{1340}{67} = 20$ cm

Hence, the height of the box is 20 cm.

Question. 95 A cuboidal tin' box opened at the top has dimensions 20 cm x 16 cm x 14 cm. What is the total area of metal sheet required to make 10 such boxes?
solution.

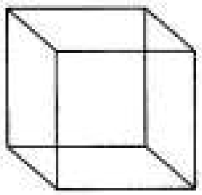
Dimensions of cuboidal tin box are20 cm x 16cm x 14 cm,

∴Area of metal sheet for 1 box = Surface area of cuboid
= 2(lb + bh+hl)
= 2(20 x 16+16 x 14+ 14 x 20)
= 2(320 + 224 + 280)
= 2(824)
= 1648 cm^2
∴ Area of metal sheet required to make 10 such boxes = 10 x 1648= 16460cm^2

Question. 96 Find the capacity of water tank, in litres, whose dimensions are 4.2 m, 3 m and 1.8 m?
solution.
Dimensions of the water tank are 4.2 m, 3 m and 1.8 m.
Capacity of water tank = Length × Breadth × Height
$= 4.2 \times 3 \times 1.8$
$= 22.68 \text{ m}^3$
∴Capacity of the water tank in litres = 22.68 × 1000 = 22680 L [∵ $1 \text{ m}^3 = 1000$ L]

Question. 97 How many cubes each of side 0.5 cm are required to build a cube of volume 8 cm^3?
solution.
Volume of a cube = $(\text{Side})^3$
∵ Side of cube = 0.5 cm
∴ Volume of the cube = $(0.5)^3 = 0.125 \text{ cm}^3$
The number of cubes required to make volume of 8 cm^3 cube
$$= \frac{8}{0.125}$$
$$= \frac{8000}{125} = 64 \text{cubes}$$

Question. 98 A wooden box (including the lid) has external dimensions
40 cm x 34 cm x 30 cm. If the wood is 1 cm thick, how many cm^3 of wood is used in it?
solution.
External dimensions of wooden box are 40 cm × 34 cm × 30 cm.



Since, the wood is 1 cm thick, it means the internal dimensions will be
(40 − 2) cm × (34 − 2) cm × (30 − 2) cm = 38 cm × 32 cm × 28 cm
∴ Wood used for the box
= Volume of the wooden box with external dimensions
− Volume of the wooden box with internal dimensions
$= 40 \times 34 \times 30 - 38 \times 32 \times 28$
$= 40800 - 34048$
$= 6752 \text{ cm}^3$

Question.99 A river 2 m deep and 45 m wide is flowing at the rate of 3 km per hour. Find the amount of water in cubic metres that runs into the sea per minute.
solution.

Depth of the river = 2 m
Width of the river = 45 m
Flowing rate of the water = 3 km/h

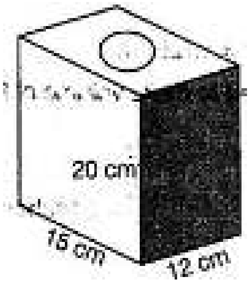
$= 3 \times \frac{1000}{60} = \frac{3000}{60}$ [∵ 1 km = 1000 m and 1 h = 60 min]

$= \frac{300}{6} = 50$ m/min

The amount of water into sea per minute

= Depth × Width × Length of water of 1 min
$= 2 \times 45 \times 50$
$= 4500 \text{ m}^3/\text{min}$

Question.100 Find the area to be painted in the following block with cylindrical hole. Given that, length is 15cm, width 12cm, height 20cfn and radius of the hole 2.8 Cm.



solution.

Lenght of the given figure = 15 cm
Width of the given figure = 12 cm
Height of the given figure = 20 cm
Radius of the hole = 2.8 cm

∴ The area to be painted = Surface area of the figure − 2 × Area of the circular hole

$= 2(lb + bh + lh) - 2\pi r^2$

$= 2(15 \times 12 + 12 \times 20 + 15 \times 20) - 2 \times \frac{22}{7} \times 2.8 \times 2.8$

$= 2(180 + 240 + 300) - \frac{44 \times 2.8 \times 2.8}{7}$

$= 2 \times 720 - 49.28$

$= 1440 - 49.28$

$= 1390.72 \text{ cm}^2$

Question. 101 A truck carrying 7.8 m3 concrete arrives at a job site. A platform of width 5 m and height 2 m is being constructed at the site. Find the length of the platform, constructed from the amount of concrete on the truck?

solution.

Total volume of concrete = 7.8 m^3
Width of the platform= 5 m
Height of the platform = 2 m
Let the length of the platform = x m
According to the question,
Volume of concrete = Volume used to make platform

∴ $7.8 = 5 \times 2 \times x$

⇒ $7.8 = 10x$ [∵ volume = length × breadth × height]

⇒ $10x = 7.8$

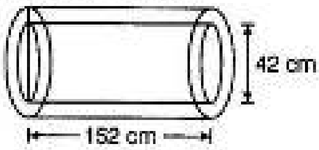
⇒ $x = 0.78$ m

Hence, the length of the platform is 0.78 m.

Question. 102 A hollow garden roller of 42 cm diameter and length 152 cm is made of cast iron 2 cm thick. Find the volume of iron used in the roller.

solution.

Diameter of the hollow garden roller = 42 cm



$\therefore$ Inner radius $= \frac{42}{2} = 21$ cm $\quad \left[\because \text{radius} = \frac{\text{diameter}}{2}\right]$

$\because$ Thickness of cast iron = 2 cm

$\therefore$ Outer radius = 21 + 2 = 23 cm

Volume of hollow cylinder $= \pi (R^2 - r^2) \times h$ [where, R is outer-radius and r is inner-radius]

$$= \frac{22}{7} \times [(23)^2 - (21)^2] \times 152$$

$$= \frac{22}{7} \times (529 - 441) \times 152$$

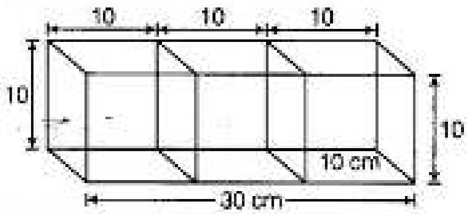
$$= \frac{22}{7} \times 88 \times 152 = \frac{22 \times 88 \times 152}{7} = \frac{294272}{7}$$

$$= 42038.85 \text{ cm}^3$$

Question. 103 Three cubes each of side 10 cm are joined end to end. Find the surface area of the resultant figure.

solution.

If three cubes each of side 10 cm are joined, then a cuboid will be formed of dimensions 30 cm × 10 cm × 10 cm.

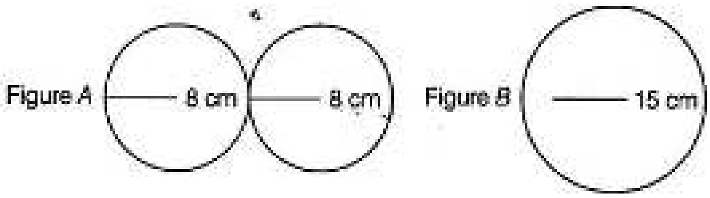


$\therefore$ Surface area of the cuboid $= 2[lb + bh + hl]$

$$= 2[30 \times 10 + 10 \times 10 + 30 \times 10]$$

$$= 2[300 + 100 + 300] = 2[700] = 1400 \text{ cm}^2$$

Question. 104 Below are the drawings of cross sections of two different pipes used to fill swimming pools. Figure A is a combination of 2 pipes each having a radius of 8 cm. Figure B is a pipe having a radius of 15 cm. If the force of the flow of water coming out of the pipes is the same in both the cases, which will fill the swimming pool faster?



solution.

In figure A, 2 pipes each having a radius of 8 cm.

$\because$ Area of a circle $= \pi r^2$

$\therefore$ Area of one pipe $= \frac{22}{7} \times 8 \times 8$

$$= \frac{22 \times 8 \times 8}{7} = \frac{22 \times 64}{7} \text{ cm}^2$$

Area of 2 pipes $= \frac{2 \times 1408}{7} = \frac{2816}{7} \text{ cm}^2 = 402.28 \text{ cm}^2$

In figure B, a pipe having radius of 15 cm.

$\therefore$ Area of the pipe $= \pi r^2$

$$= \frac{22}{7} \times 15 \times 15 = \frac{22}{7} \times 225$$

$$= \frac{4950}{7} = 707.14 \text{ cm}^2$$

Clearly, the surface area of pipe B is greater. So, pipe B fill the swimming pool faster.

Question. 105 A swimming pool is 200 m x 50 m and has an average depth of 2 m. By the end of a summer day, the water level drops by 2 cm. How many cubic metres of water is lost on the day?

solution.

Dimensions of swimming pool are 200 m × 50 m.

Average depth of the swimming pool = 2 m

At the end of summer day the water level drops by 2 cm.

$\therefore$ Volume of water in swimming pool = Length × Breadth × Depth

$$= 200 \times 50 \times 2 = 20000 \text{ m}^3$$

If water level drops by 2 cm, it means new level of water

$$= \left(2 - \frac{2}{100}\right) \text{m} = 1.98 \text{ m} \quad \left[\because 1 \text{ cm} = \frac{1}{100} \text{ m}\right]$$

Volume of water after summer day $= 200 \times 50 \times 1.98$

$$= 19800 \text{ m}^3$$

So, water in cubic metres was lost on that day

= Initial volume − Volume after summer day

$$= 20000 - 19800 = 200 \text{ m}^3$$

Question. 106 A housing society consisting of 5500 peoples needs 100 L of water per person per day. The cylindrical supply tank is 7 m high and has a diameter 10 m. For how many days will the water in the tank last for
the society?

solution.

Total number of peoples = 5500

Water required per person per day = 100 L

Total requirement of water by 5500 peoples

$$= 100 \times 5500 = 550000 \text{ L}$$

Height of the cylindrical tank = 7 m

Diameter of the cylindrical tank = 10 m

$\therefore$ Radius = 5 m $\quad \left[\because \frac{\text{diameter}}{2} = \text{radius}\right]$

$\therefore$ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 7$

$$= 22 \times 25 = 550 \text{ m}^3$$

$$= 550 \times 1000 = 550000 \text{ L} \quad [\because 1 \text{ m}^3 = 1000 \text{ L}]$$

Hence, for 1 day the water in the tank lost for the society and in one day society needs 550000 L of water.

Question. 107 Metallic discs of radius 0.75 cm and thickness 0.2 cm are melted to obtain 508.68 cm^3 of metal. Find the number of disc melted (Use π = 3.14)

Solution.

Radius of metallic disc = 0.75 cm

Thickness of disc = 0.2 cm

Total volume of material which will be used in forming/melting of disc = 508.68 cm^3

$\therefore$ Material required for one disc = Volume of cylinder

$$= \pi r^2 h = \frac{22}{7} \times 0.75 \times 0.75 \times 0.2 \text{ [}\because \text{ shape of a disc is a cylinder]}$$

$$= 3.14 \times 0.75 \times 0.75 \times 0.2$$

$$= 0.35325 \text{ cm}^3$$

$$\text{Number of discs can be melted} = \frac{\text{Total volume of metal obtained after melting}}{\text{Volume of one disc}}$$

$$= \frac{508.68}{0.35325} = 1440 \text{ discs}$$

Question. 108 The ratio of the radius and height of a cylinder is 2:3. If its volume is 12936 cm^3 find the the total surface area of the cylinder.

solution.

The ratio of the radius and height of a cylinder = 2 : 3

Let the radius of the cylinder be $2x$ and the height of the cylinder be $3x$.

Volume of the cylinder = 12936 cm^3

$\because$ Volume of a cylinder = $\pi r^2 h$

$\therefore \quad 12936 = \frac{22}{7} \times (2x)^2 \times 3x$

$\Rightarrow \quad 12936 = \frac{22}{7} \times 4x^2 \times 3x$

$\Rightarrow \quad 12936 = \frac{264}{7} x^3$

$\Rightarrow \quad x^3 = \frac{12936 \times 7}{264} = 49 \times 7$

$\Rightarrow \quad x^3 = 7 \times 7 \times 7 = (7)^3$

$\Rightarrow \quad x^3 = (7)^3$

$\therefore \quad x = 7$

So, radius = $2x = 2 \times 7 = 14$ cm and height = $3x = 3 \times 7 = 21$ cm

The total surface area of the cylinder = $2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 14 (14 + 21)$$

$$= \frac{44 \times 14}{7} \times 35 = 44 \times 14 \times 5 = 3080 \text{ cm}^2$$

Question. 109 External dimensions of a closed wooden box are in the ratio 5:4:3. If the cost of painting its outer surface at the rate of Rs 5 per dm^2 is Rs 11750, find the dimensions of the box.

solution.

External dimensions of a closed wooden box are in the ratio 5 : 4 : 3.

Let the external dimensions of the closed wooden box be $5x$, $4x$ and $3x$.

The cost of painting = ₹ 5 per dm^2

Total cost of painting = ₹ 11750

$$\therefore \text{Total surface area} = \frac{\text{Total cost of painting}}{\text{Cost of painting per dm}^2} = \frac{11750}{5}$$

$$= 2350 \text{ dm}^2$$

Total surface area of a cuboid

$$= 2(lb + bh + hl)$$

$= 2(5x \times 4x + 4x \times 3x + 3x \times 5x)$

$= 2(20x^2 + 12x^2 + 15x^2)$

$= 2 \times 47x^2 = 94x^2$

Since, total surface area = 2350 dm^2

$\Rightarrow \quad 94x^2 = 2350$

$\Rightarrow \quad x^2 = \frac{2350}{94} = 25$

$\therefore \quad x = 5$

Hence, dimensions of the box are $5x \times 5 = 25$ dm, $4x \times 5 = 20$ dm and $3x \times 5 = 15$ dm.

Question. 110 The capacity of a closed cylindrical vessel of height 1 m is 15.4 L. How many square metres of metal sheet would be needed to make it?

solution.

Height of cylindrical vessel = 1 m

Capacity of the cylindrical vessel = 15.4 L

In metre cube $= \frac{15.4}{1000} = .0154 \, m^3$ $\quad [\because 1 \, m^3 = 1000 \, L]$

Volume of a cylinder $= \pi r^2 h$

$\Rightarrow \quad \frac{22}{7} \times r^2 \times 1 = 0.0154$

$\Rightarrow \quad r^2 = \frac{0.0154}{3.14} = 0.0049$

$\Rightarrow \quad r = \sqrt{0.0049} = 0.07$ m

$\therefore$ Metal of sheet required $= 2\pi rh = 2 \times \frac{22}{7} \times 0.07 \times 1 = 0.4396 = 0.44 \, m^2$

Question. 111. What will happen to the volume of the cube, if its edge

(a) tripled (b) reduced to one-fourth?

solution.

Let each side of the cube be a, then its volume $= a^3$ $\quad [\because \text{volume of a cube} = (\text{side})^3]$

(a) If side became triple, then volume will be $= (3a)^3 = 27a^3$

Hence, new volume of the cube will 27 times of original volume of the cube.

(b) If side reduced to one fourth $= a \times \frac{1}{4} = \frac{a}{4}$

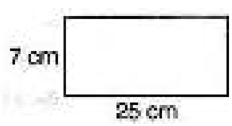
Now, its volume $= \left(\frac{a}{4}\right)^3 = \frac{a^3}{64}$

Hence, new volume $\frac{1}{64}$ times of original volume.

Question. 112 A rectangular sheet of dimensions 25 cm x 7 cm is rotated about its . longer side. Find the volume and the whole surface area of the solid thus generated.

solution.

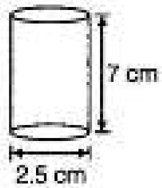
A rectangular sheet of demensions 25 cm × 7 cm is rotated about its longer side which makes a cylinder with base 25 cm /and height 7 cm.



Surface area of a base = $2\pi r$

$\therefore \quad 2\pi r = 25 \text{ cm}$

$\Rightarrow \quad r = \frac{25 \times 7}{2 \times 22} = \frac{175}{44} \text{ cm}$



Volume of a cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{175}{44} \times \frac{175}{44} \times 7$

$= \frac{175 \times 175}{2 \times 44} = \frac{30625}{88}$

$= 348.011 \text{ cm}^3$

Surface area = $2\pi rh = 2 \times \frac{22}{7} \times \frac{175}{44} \times 7$

$= \frac{44}{44} \times 175$

$= 175 \text{ cm}^2$

Question. 113 From a pipe of inner radius 0.75 cm, water flows at the rate of 7 m per second. Find the volume in litres of water delivered by the pipe in 1 h.

solution.

Radius of pipe = 0.75 cm = 0.0075 m $\left[\because 1 \text{ cm} = \frac{1}{100} \text{m}\right]$

The rate of water flow = 7 m/s

$\therefore$ Length of water in 1 sec = 7 m

$\therefore$ Volume of water flow in 1 hour $= 60 \times 60 \times \frac{22}{7} \times 0.0075 \times 0.0075 \times 7$

$= 3600 \times 3.14 \times 0.00005625 \times 7$

$= 11304 \times 7 \times 0.00005625$

$= 79128 \times 0.00005625$

$= 4.45095 \text{ m}^3$

$\simeq 4.45000 \text{ m}^3$

$\simeq 4450000 \text{ cm}^3 \simeq 4450 \text{ L}$ $[\because 1000 \text{ cm}^3 = 1 \text{ L}]$

Question. 114 Four times the area of the curved surface of a cylinder is equal to 6 times the sum of the areas of its bases. If its height is 12 cm, find its curved surface area.

solution.

Let the radius and height of the cylinder be r and h, respectively.

Curved surface area of cylinder = 2 πrh

Area of base = πr^2

Sum of areas of bases = $2\pi r^2$

According to the question,

4 × Curved surface area = 6 × Sum of areas of bases

$4 \times 2\pi rh = 6 \times 2\pi r^2$

$\Rightarrow \quad 8\pi rh = 12\pi r^2$

$\Rightarrow \quad 2h = 3r$

$\Rightarrow \quad r = \frac{2}{3} h$

$\therefore \quad r = \frac{2}{3} \times 12 = 8 \text{ cm}$ $[\because h = 12 \text{ cm, given}]$

$\therefore$ Curved surface area of the cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 8 \times 12 = \frac{44 \times 8 \times 12}{7}$

$= 603.428 \text{ cm}^2$

Question. 115 A cylindrical tank lias a radius of 154 cm. It is filled with water to a height of 3 m. If water to a height of 4.5 m is poured into it, what will be the increase in the volume of water in kL?

solution.

Radius of cylindrical tank = 154 cm

Initial height of water tank = 3 m = 3 × 100 cm [$\because$ 1 m = 100 cm]

$\therefore$ Volume of water = $\pi r^2 h = \frac{22}{7} \times 154 \times 154 \times 3 \times 100$

$= 22360800 \text{ cm}^3$

If height of water 4.5 m is poured into it, then volume of water

$= \frac{22}{7} \times 154 \times 154 \times 4.5 \times 100$

$= 33541200 \text{ cm}^3$

Increase in volume = 33541200 − 22360800

$= 11180400 \text{ cm}^3$

= 11180.4 L [$1000 \text{ cm}^3 = 1 \text{L}$]

= 11.1804 kL [1 kL = 1000 L]

Question. 116 The length, breadth and height of a cuboidal reservoir is 7 m, 6 m and 15 m respectively. 8400 L of water is pumped out from the reservoir. Find the fall in the water level in the reservoir.

solution.

Length of a cuboidal reservoir = 7 m

Breadth of a cuboidal reservoir = 6 m

Height of a cuboidal reservoir = 15 m

Capacity of cuboidal reservoir = $l \times b \times h = 7 \times 6 \times 15 = 630 \text{ m}^3$

If 8400 L of water is pumped out.

In meter cubic = $\frac{8400}{1000} = 8.4 \text{ m}^3$

Fall in water level = $630 - 8.4 = 621.6 \text{ m}^3$

= 621.6 × 1000 [$\because 1 \text{ m}^3 = 1000$ L]

= 621600 L

Question. 117 How many bricks of size 22 cm x 10 cm x 7 cm are required to construct a wall 11 m long, 3.5 m high and 40 cm thick, if the cement and sand used in the construction occupy (1/10)th part of the wall?

solution.

Volume of each brick = 22 cm × 10 cm × 7 cm

$= 1540 \text{ cm}^3 = 0.00154 \text{ m}^3$

Volume of wall = $l \times b \times h = 11 \text{ m} \times 3.5 \text{ m} \times \frac{40}{100} \text{ m}$ [$\because$ 1 m = 100 cm]

$= 11 \times 3.5 \times 0.4 = 15.4 \text{ m}^3$

If 1/10th part of the wall used in cement and sand, then part of wall used by cement and sand remaining = $\frac{15.4}{10} \text{m}^3 = 1.54 \text{ m}^3$

Remaining part = $15.4 - 1.54 = 13.86 \text{ m}^3$

Number of bricks = $\frac{\text{Volume of wall to be construct}}{\text{Volume of each brick}} = \frac{13.86}{0.00154} = 9000$

Question. 118 A rectangular examination hall having seats for 500 candidates has to be built so, as to allow 4 cubic metres of air and 0.5 square metres of floor area per candidate. If the length of hall be 25 m, find the height and breadth of the hall.

solution.

Total number of seats in rectangular examination hall = 500

Length of the hall = 25 m

Cubic metres of air for per candidates = 4 m^3

Square metres of floor area for per candidate = 0.5 m^2

$\therefore$ Height of the hall = $\dfrac{\text{Volume of air per candidate}}{\text{Square metre of floor area for one candidate}} = \dfrac{4}{0.5} = \dfrac{40}{5} = 8$ m

Total capacity of the hall = 500 × 4 = 2000 m^3.

$\therefore$ Breadth of the hall = $\dfrac{2000}{25 \times 8} = \dfrac{80}{8} = 10$ m

Question. 119 The ratio between the curved surface area and the total surface area of ' a right circular cylinder is 1: 2. Find the ratio between the height and radius of the cylinder.

solution.

Curved surface area of a cylinder = $2\pi rh$

Total surface area of a cylinder = $2\pi r(r + h)$

The ratio between the curved surface area and the total surface area of a cylinder is 1 : 2.

$$\therefore \quad \frac{1}{2} = \frac{2\pi rh}{2\pi r(r+h)}$$

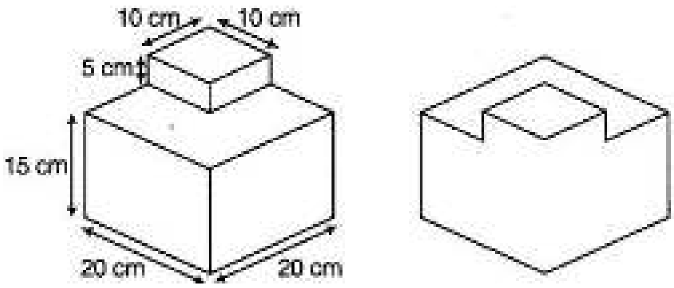
$$\Rightarrow \quad \frac{1}{2} = \frac{h}{(r+h)}$$

$$\Rightarrow \quad 2h = (r + h)$$

$$\Rightarrow \quad 2h - h = r \Rightarrow h = r$$

$\therefore$ Ratio of height and radius = 1 : 1.

Question. 120 A birthday cake has two tiers as shown in the figure below. Find the volume of the cake.



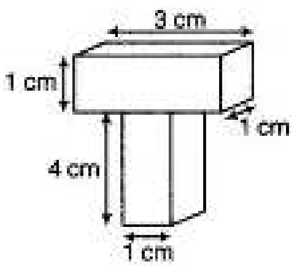
solution.

Volume of the cake = Volume of lower cuboid + Volume of upper cuboid

$= 20 \times 20 \times 15 + 10 \times 10 \times 5$ [$\because$ volume of cuboid = $l \times b \times h$]

$= 6000 + 500 = 6500 \text{ cm}^3$

Question. 121



Solution.

Surface area of the figure

$= 2[3 \times 1 + 1 \times 1 + 3 \times 1] + 2[4 \times 1 + 1 \times 1 + 4 \times 1] - [1 \times 1 + 1 \times 1]$

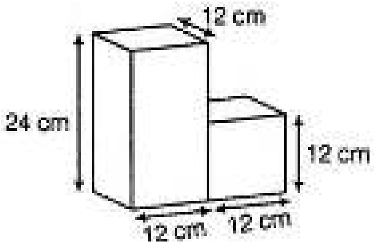
$= 2[3 + 1 + 3] + 2[4 + 1 + 4] - [1 + 1]$

$= 2[7] + 2[9] - [2] = 14 + 18 - 2$

$= 32 - 2$

$= 30 \text{ cm}^2$

Question. 122



Solution.

In the given figure, there are two cuboides are joined.

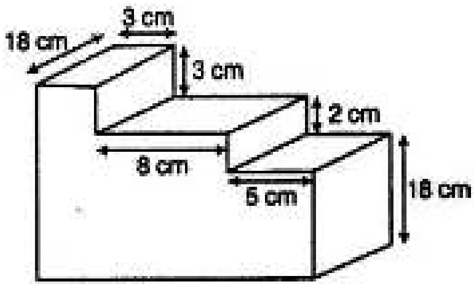
∴ Total surface area of the given solid

$= 2(24 \times 12 + 12 \times 12 + 12 \times 24) + 2(12 \times 12 + 12 \times 12 + 12 \times 12) - 2(12 \times 12)$

$= 1440 + 864 - 288$

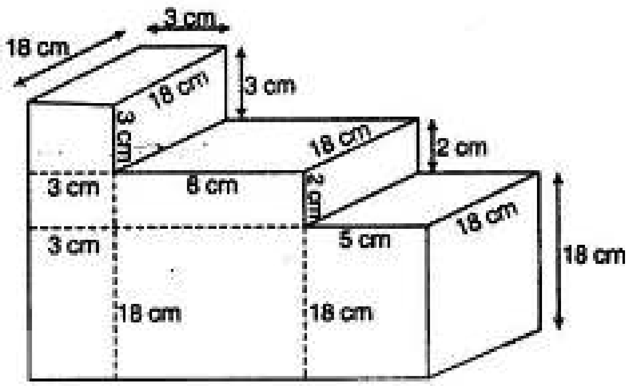
$= 2016 \text{ cm}^2$

Question. 123



Solution.

To find the total surface area, we draw the figure as given below.



∴ Upper surface area

$= 18 \times 3 + 8 \times 18 + 5 \times 18$

$= 54 + 144 + 90$

$= 288 \text{ cm}^2$

∴ Lower surface area $= 288 \text{ cm}^2$

∴ Surface area of thosse faces which are flat from right

$= 18 \times 18 + 2 \times 18 + 3 \times 18$

$= 324 + 36 + 54$

$= 414 \text{ cm}^2$

Also, surface area of that face which are flat from left

$$= 414 \text{ cm}^2$$

Surface area of from face

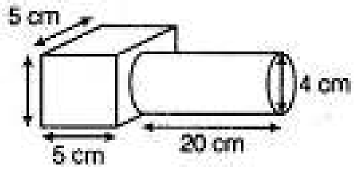
$$= 18 \times 5 + 2 \times 8 + 8 \times 18 + 3 \times 2 + 3 \times 18 + 3 \times 3$$
$$= 90 + 16 + 144 + 6 + 54 + 9$$
$$= 319 \text{ cm}^2$$

Surface area of the back face = 319 cm^2

∴ Total surface area

$$= 288 + 288 + 414 + 414 + 319 + 319$$
$$= 288 + 1754$$
$$= 2042 \text{ cm}^2$$

Question. 124



Solution.

In the given figure, there is a cube of side 5 cm and a cylinder of hight 20 cm and radius is 2 cm.

So, total surface area = Surface area of the cube + Curved surface area of cylinder

$$= 6(5)^2 + 2\pi(2) \times 20 \quad \left[\begin{array}{l}\because \text{surface area of cube} = 6a^2 \\ \therefore \text{surface area of cylinder} = 2\pi rh\end{array}\right]$$

$$= 150 + 80\pi$$

Total surface area = $150 + 251.2 = 401.2 \text{ cm}^2$

Note *In this question assume that cylinder is at end open.*

Question. 125 Water flows from a tank with a rectangular base measuring 80 cm x 70 cm into another tank with a square base of side 60 cm. If the water in the first tank is 45 cm deep, how deep will it be in the second tank?

solution.

Dimensions of rectangular base tank are 80 cm × 70 cm.

Height of rectangle base tank = 45 cm

Each side of square base tank = 60 cm

Let h be the height of square base tank.

Volume of rectangular tank = Volume of square tank

$$\Rightarrow \quad 80 \times 70 \times 45 = 60 \times 60 \times h \quad [\because \text{volume of cuboidal} = l \times b \times h]$$

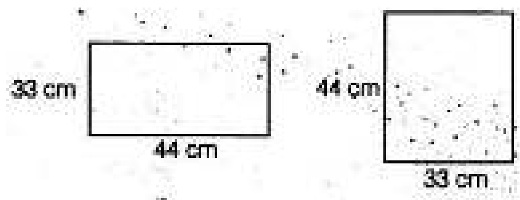
$$\Rightarrow \quad \frac{80 \times 70 \times 45}{60 \times 60} = h$$

$$\therefore \quad h = 70 \text{ cm}$$

Hence, water in second tank will be 70 cm deep.

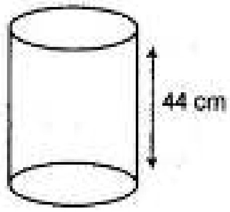
Question. 126 A rectangular sheet of paper is rolled in two different ways to form two different cylinders. Find the volume of cylinders in each case if the sheet measures 44 cm x 33 cm.

solution.



(i) (ii)

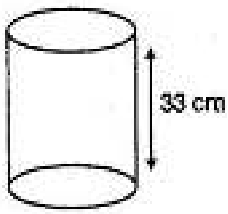
In Ist case,
If sheet rolled along 44 cm



$\Rightarrow$ $$2\pi r = 44\text{ cm}$$

$\therefore$ $$r = \frac{44}{2\pi} = \frac{44}{2} \times \frac{7}{22} = 7\text{ cm}$$

In IInd case,
It sheet rolled along 33 cm.



$\Rightarrow$ $$2\pi r = 33$$

$\therefore$ $$r = \frac{33}{2\pi} = \frac{33 \times 7}{2 \times 22}$$

$$= \frac{3 \times 7}{2 \times 2} = \frac{21}{4}\text{ cm}$$