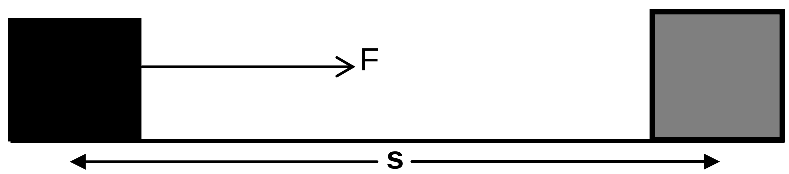
WORK ENERGY AND POWER

WORK

PHYSICAL DEFINITION

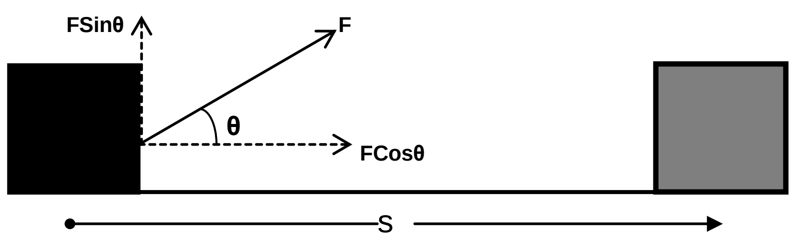
When the point of application of force moves in the direction of the applied force under its effect then work is said to be done.

MATHEMATICAL DEFINITION OF WORK



Work is defined as the product of force and displacement in the direction of force

$$W = F \times s$$

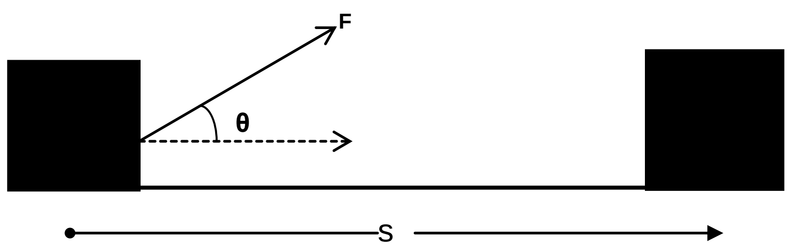


If force and displacement are not parallel to each other rather they are inclined at an angle, then in the evaluation of work component of force (F) in the direction of displacement (s) will be considered.

$$W = (F\cos\theta) \times s$$

or, $$W = Fs\cos\theta$$

VECTOR DEFINITION OF WORK



Force and displacement both are vector quantities but their product, work is a scalar quantity, hence work must be scalar product or dot product of force and displacement vector.

$$\mathbf{W = \vec{F} \cdot \vec{s}}$$

WORK DONE BY VARIABLE FORCE

Force varying with displacement

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

$$dW = \vec{F} \cdot \vec{ds}$$

$$\mathbf{W = \int_{s_1}^{s_2} \vec{F} \cdot \vec{ds}}$$

Force varying with time

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated.

$$dW = \vec{F} \cdot \vec{ds}$$

Multiplying and dividing by dt,

$$dW = \frac{\vec{F} \cdot \vec{ds}\ dt}{dt}$$

or,

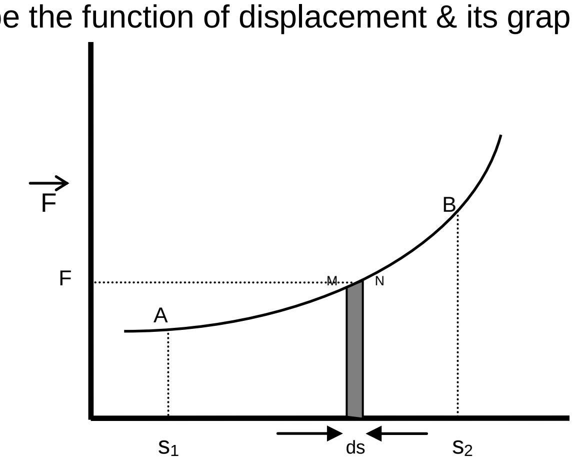
$$dW = \vec{F} \cdot \vec{v}\ dt \quad (v=ds/dt)$$

Total work is obtained by integrating the elementary work from initial to final limits.

$$\mathbf{W = \int_{t_1}^{t_2} \vec{F} \cdot \vec{v}\, dt}$$

WORK DONE BY VARIABLE FORCE FROM GRAPH

Let force be the function of displacement & its graph be as shown.



To find work done from s_1 to s_2 we consider two points M & N very close on the graph such that magnitude of force (F) is almost same at both the points. If elementary displacement from M to N is ds, then elementary work done from M to N is.

$$dW = F.ds$$
$$dW = (\text{length x breadth})\text{of strip MNds}$$
$$dW = \text{Area of strip MNds}$$

Thus work done in any part of the graph is equal to area under that part. Hence total work done from s_1 to s_2 will be given by the area enclosed under the graph from s_1 to s_2.

$$\mathbf{W = Area\ (ABS_2S_1A)}$$

DIFFERENT CASES OF WORK DONE BY CONSTANT FORCE

Case i) Force and displacement are in same direction

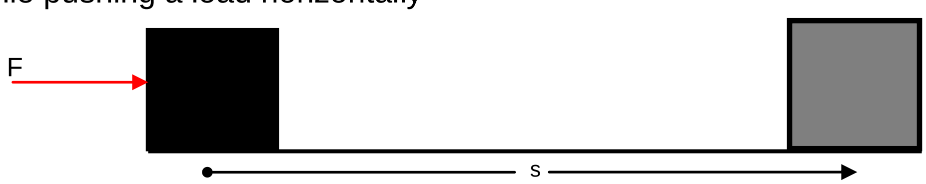
$\theta = 0$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 0$

or, $\mathbf{W = Fs}$

Ex - Coolie pushing a load horizontally



Case ii) Force and displacement are mutually perpendicular to each other

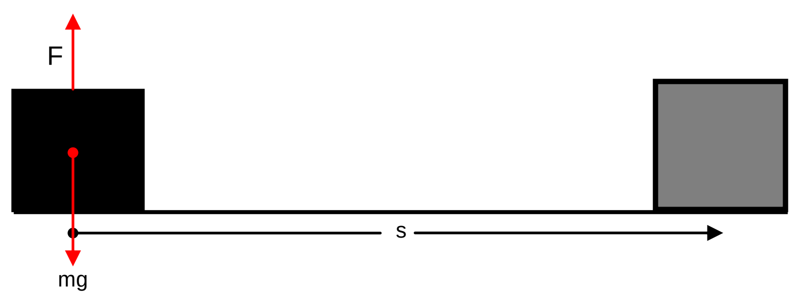
$\theta = 90$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 90$

or, $\mathbf{W = 0}$

Ex - coolie carrying a load on his head & moving horizontally with constant velocity. Then he applies force vertically to balance weight of body & its displacement is horizontal.



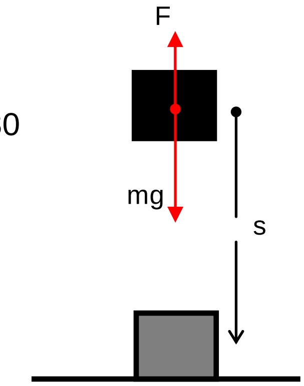
(3) Force & displacement are in opposite direction

$\theta = 180$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 180$

or, $\mathbf{W = - Fs}$



Ex - Coolie carrying a load on his head & moving vertically down with constant velocity. Then he applies force in vertically upward direction to balance the weight of body & its displacement is in vertically downward direction.

ENERGY

Capacity of doing work by a body is known as energy.

Note - Energy possessed by the body by virtue of any cause is equal to the total work done by the body when the cause responsible for energy becomes completely extinct.

TYPES OF ENERGIES

There are many types of energies like mechanical energy, electrical, magnetic, nuclear, solar, chemical etc.

MECHANICAL ENERGY

Energy possessed by the body by virtue of which it performs some mechanical work is known as mechanical energy.

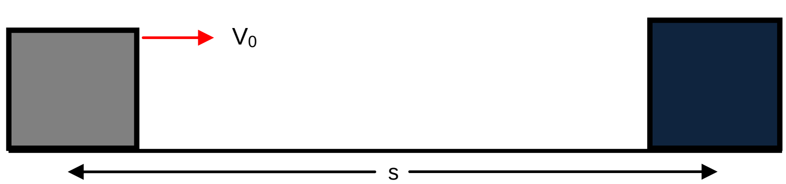
It is of basically two types-

(i) Kinetic energy

(ii) Potential energy

KINETIC ENERGY

Energy possessed by body due to virtue of its motion is known as the kinetic energy of the body. Kinetic energy possessed by moving body is equal to total work done by the body just before coming out to rest.



Consider a body of man (m) moving with velocity (v_o).After travelling through distance (s) it comes to rest.

$$u = v_o$$
$$v = 0$$
$$s = s$$

Applying, $v^2 = u^2 + 2as$

$$0 = v_0^2 + 2as$$

or, $2as = -v_0^2$

or, $a = \frac{-v_o^2}{2s}$

Hence force acting on the body,

$$F = ma$$
$$F_{on\ body} = -\frac{mv_o^2}{2s}$$

But from Newton's third law of action and reaction, force applied by body is equal and opposite to the force applied on body

$$F_{by\ body} = -F_{on\ body}$$
$$= +\frac{mv_o^2}{2s}$$

Therefore work done by body,

$$W = \vec{F}.\vec{s}$$

or, $W = \frac{mv_0^2}{2s}.s.\text{Cos } 0$

or, $W = \frac{1}{2} mv_o^2$

Thus K.E. stored in the body is,

$$\mathbf{K.E. = \frac{1}{2} mv_o^2}$$

KINETIC ENERGY IN TERMS OF MOMENTUM

K.E. of body moving with velocity v is

$$K.E. = \frac{1}{2} mv_o^2$$

Multiplying and dividing by m

$$K = \frac{1}{2}\frac{mv^2 \times m}{m}$$
$$= \frac{1}{2}\frac{m^2v^2}{m}$$

But, mv = p (linear momentum)

Therefore, $\mathbf{K = \frac{p^2}{2m}}$

POTENTIAL ENERGY

Energy possessed by the body by virtue of its position or state is known as potential energy. Example:- gravitational potential energy, elastic potential energy, electrostatic potential energy etc.

GRAVITATIONAL POTENTIAL ENERGY

Energy possessed by a body by virtue of its height above surface of earth is known as gravitational potential energy. It is equal to the work done by the body situated at some height in returning back slowly to the surface of earth.

Consider a body of mass m situated at height h above the surface of earth. Force applied by the body in vertically downward direction is

$$F = mg$$

Displacement of the body in coming back slowly to the surface of earth is

$$s = h$$

Hence work done by the body is

$$W = FsCos\theta$$

or, $$W = FsCos0$$

or, $$W = mgh$$

This work was stored in the body in the form of gravitational potential energy due to its position. Therefore

$$\textbf{G.P.E = mgh}$$

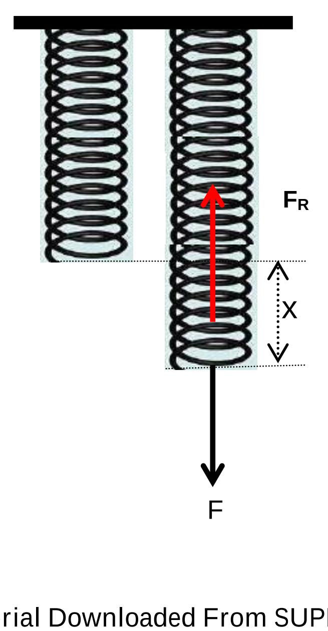
ELASTIC POTENTIAL ENERGY

Energy possessed by the spring by virtue of compression or expansion against elastic force in the spring is known as elastic potential energy.

Spring

It is a coiled structure made up of elastic material & is capable of applying restoring force & restoring torque when disturbed from its original state. When force (F) is applied at one end of the string, parallel to its length, keeping the other end fixed, then the spring expands (or contracts) & develops a restoring force (F_R) which balances the applied force in equilibrium.

On increasing applied force spring further expands in order to increase restoring force for balancing the applied force. Thus restoring force developed within the spring is directed proportional to the extension produced in the spring.



$$F_R \propto x$$

or, $$\mathbf{F_R = kx}$$ (k is known as spring constant or force constant)

If $x = 1$, $F_R = k$

Hence force constant of string may be defined as the restoring force developed within spring when its length is changed by unity.

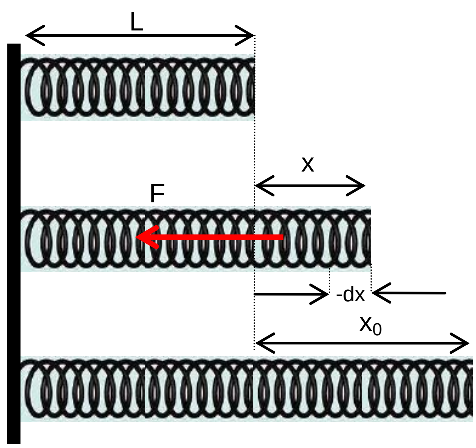
But in equilibrium, restoring force balances applied force.

$$\mathbf{F = F_R = k\,x}$$

If $x = 1$, $F = 1$

Hence force constant of string may also be defined as the force required to change its length by unity in equilibrium.

Mathematical Expression for Elastic Potential Energy



Consider a spring of natural length 'L' & spring constant 'k' its length is increased by x_o. Elastic potential energy of stretched spring will be equal to total work done by the spring in regaining its original length.

If in the process of regaining its natural length, at any instant extension in the spring was x then force applied by spring is

$$F = kx$$

If spring normalizes its length by elementary distance dx opposite to x under this force then work done by spring is

$$dW = F.\,(-dx)\,.\,\text{Cos}0$$

(force applied by spring F and displacement –dx taken opposite to extension x are in same direction)

$$dW = -kxdx$$

Total work done by the spring in regaining its original length is obtained in integrating dW from x_0 to 0

$$W = \int_{x_0}^{0} -kx\,dx$$

or, $$W = -k\left[x^2/2\right]_{x_0}^{0}$$

or, $$W = -k\,(0^2/2 - x_0^2/2)$$

o r, $$W = -k\,(0 - x_0^2/2)$$

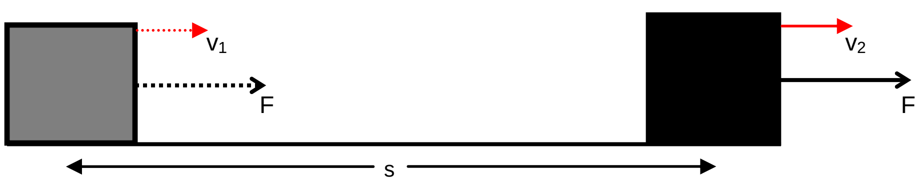
or, $$W = \frac{1}{2} k x_o^2$$

This work was stored in the body in the form of elastic potential energy.

$$\mathbf{E.P.E = \frac{1}{2} k x_o^2}$$

WORK ENERGY THEOREM

It states that total work done on the body is equal to the change in kinetic energy.(Provided body is confined to move horizontally and no dissipating forces are operating).



Consider a body of man m moving with initial velocity v_1. After travelling through displacement s its final velocity becomes v_2 under the effect of force F.

$$u = v_1$$
$$v = v_2$$
$$s = s$$

Applying, $$v^2 = u^2 + 2as$$
$$v_2^2 = v_1^2 + 2as$$

or, $$2as = v_2^2 - v_1^2$$

or, $$a = \frac{v_2^2 - v_1^2}{2s}$$

Hence external force acting on the body is

$$F = ma$$
$$F = m\,\frac{v_2^2 - v_1^2}{2s}$$

Therefore work done on body by external force

$$W = \vec{F} \cdot \vec{s}$$

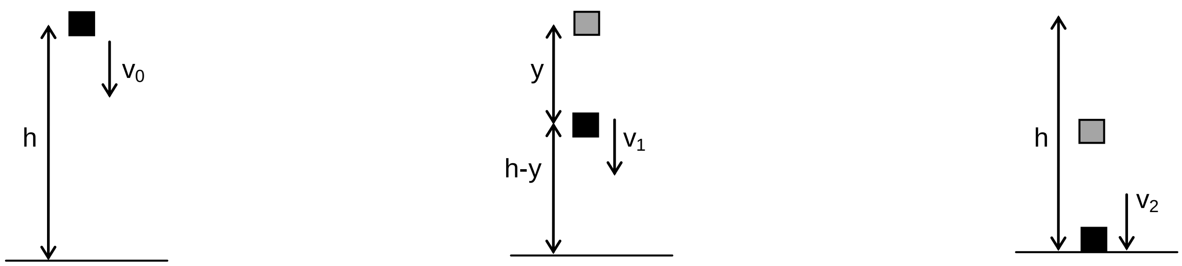
or, $$W = m\,\frac{v_2^2 - v_1^2}{2s} \cdot s\,.\mathrm{Cos}\,0$$

(since force and displacement are in same direction) or, $W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

or, $\mathbf{W = K_2 - K_1}$

or, $\mathbf{W = \Delta K}$

PRINCIPLE OF CONSERVATION OF ENERGY



It states that energy can neither be creased neither be destroyed. It can only be converted from one form to another.

Consider a body of man m situated at height h & moving with velocity v_o. Its energy will be.

$$E_1 = P_1 + K_1$$

or, $\mathbf{E_1 = mgh + \frac{1}{2} m v_o^2}$

If the body falls under gravity through distance y, then it acquires velocity v_1 and its height becomes (h-y)

$u = v_o$
$s = y$
$a = g$
$v = v_1$

From $v^2 = u^2 + 2as$

$$v_1^2 = v_o^2 + 2gy$$

Energy of body in second situation

$$E_2 = P_2 + K_2$$

or, $E_2 = mg(h-y) + \frac{1}{2} mv^2$

or, $E_2 = mg(h-y) + \frac{1}{2} m (v_o^2 + 2gy)$

or, $E_2 = mgh - mgy + \frac{1}{2} m v_o^2 + mgy$

or, $\mathbf{E_2 = mgh + \frac{1}{2} m v_o^2}$

Now we consider the situation when body reaches ground with velocity v_2

$u = v_o$
$s = h$
$a = g$
$v = v_2$

From $v^2 = u^2 + 2as$

$$_2^2 = v_o^2 + 2gh$$

Energy of body in third situation

$$E_3 = P_3 + K_3$$

or, $E_3 = mg0 + ½ mv_2^2$

or, $E_3 = 0 + ½ m (v_o^2 + 2gh)$

or, $\mathbf{E_3 = ½ mv_o^2 + mgh}$

From above it must be clear that $\mathbf{E_1 = E_2 = E_3}$. This proves the law of conservation of energy.

CONSERVATIVE FORCE

Forces are said to be conservative in nature if work done against the forces gets conversed in the body in form of potential energy. Example:- gravitational forces, elastic forces & all the central forces.

PROPERTIES OF CONSERVATIVE FORCES

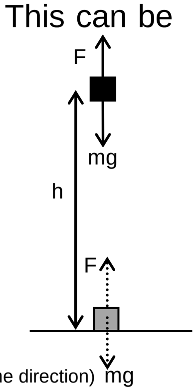
1. Work done against these forces is conserved & gets stored in the body in the form of P.E.
2. Work done against these forces is never dissipated by being converted into non-usable forms of energy like heat, light, sound etc.
3. Work done against conservative forces is a state function & not path function i.e. Work done against it, depends only upon initial & final states of body & is independent of the path through which process has been carried out.
4. Work done against conservative forces is zero in a complete cycle.

TO PROVE WORK DONE AGAINST CONSERVATIVE FORCES IS A STATE FUNCTION

Consider a body of man m which is required to be lifted up to height h. This can be done in 2 ways.

(i) By directly lifting the body against gravity

(ii) By pushing the body up a smooth inclined plane.



Min force required to lift the body of mass m vertically is

$$F = mg$$

And displacement of body in lifting is

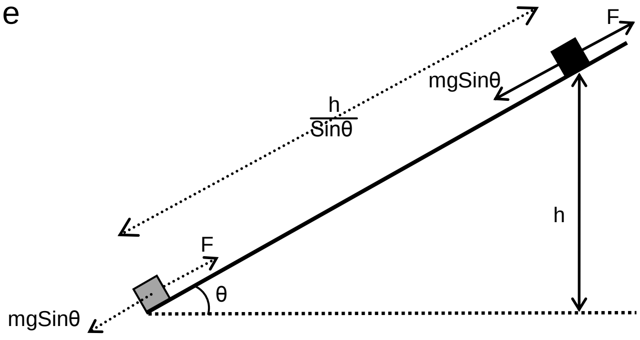
$$s = h$$

Hence work done in lifting is

$W_1 = FsCos0^o$ (since force and displacement are in same direction)

$$\mathbf{W_1 = mgh}$$

Now we consider the same body lifted through height h by pushing it up a smooth inclined plane



Min force required to push the body is

$$F = mg\sin\theta$$

And displacement of body in lifting is

$$s = \frac{h}{\sin\theta}$$

Hence work done in pushing is

$$W_2 = Fs\cos 0$$

or, $$W_2 = mg\sin\theta \cdot \frac{h}{\sin\theta} \cdot 1$$

or, $$\mathbf{W_2 = mgh}$$

From above $W_1 = W_2$ we can say that in both the cases work done in lifting the body through height 'h' is same.

To Prove That Work Done Against Conservative Forces Is Zero In A Complete Cycle

Consider a body of man m which is lifted slowly through height h & then allowed to come back to the ground slowly through height h.

For work done is slowly lifting the body up,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical up displacement of the body is

$$s = h$$

Hence work done is

$$W = FsCos\theta$$

or, $W_I = FsCos0$ (since force and displacement are in same direction)

or, $\mathbf{W_I = mgh}$ (since force and displacement are in same direction)

For work done is slowly bringing the body down,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical down displacement of the body is

$$s = h$$

Hence work done is

or, $W_2 = FsCos180$(since force and displacement are in opposite direction)

or, $\mathbf{W_2 = - mgh}$

Hence total work done against conservative forces in a complete cycle is

$$W = W_1 + W_2$$

or, $W = (mgh) + (-mgh)$

or, $\mathbf{W = 0}$

NON-CONSERVATIVE FORCES

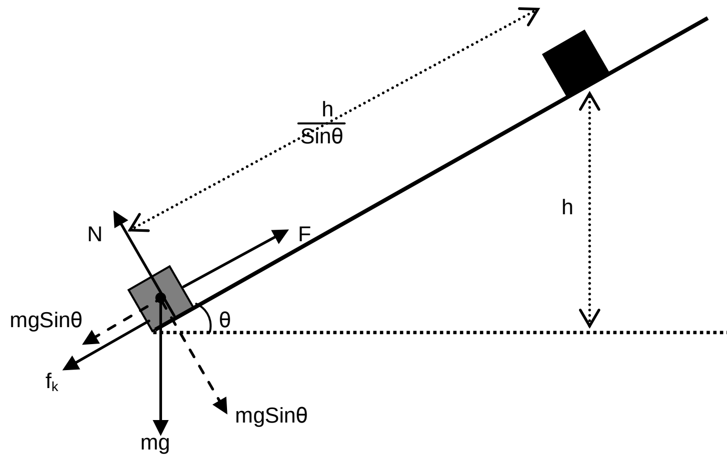
Non conservative forces are the forces, work done against which does not get conserved in the body in the form of potential energy.

PROPERTIES OF NON-CONSERVATIVE FORCES

1. Work done against these forces does not get conserved in the body in the form of P.E.
2. Work done against these forces is always dissipated by being converted into non usable forms of energy like heat, light, sound etc.
3. Work done against non-conservative force is a path function and not a state function.
4. Work done against non-conservative force in a complete cycle is not zero.

PROVE THAT WORK DONE AGAINST NON–CONSERVATIVE FORCES IS A PATH FUNCTION

Consider a body of mass (m) which is required to be lifted to height ‘h’ by pushing it up the rough incline of inclination.



Minimum force required to slide the body up the rough inclined plane having coefficient of kinetic friction μ with the body is

$$F = mgSin\theta + f_k$$

or, $$F = mgSin\theta + \mu N$$

or, $$F = mgSin\theta + \mu mgCos\theta$$

Displacement of the body over the incline in moving through height h is

$$s = \frac{h}{Sin\theta}$$

Hence work done in moving the body up the incline is

$$W = F.s.Cos0$$ (since force and displacement are in opposite direction)

or, $$W = (mgSin\theta + \mu mgCos\theta).\frac{h}{Sin\theta}.1$$

or, $$\mathbf{W = mgh + \frac{\mu mgh}{Tan\theta}}$$

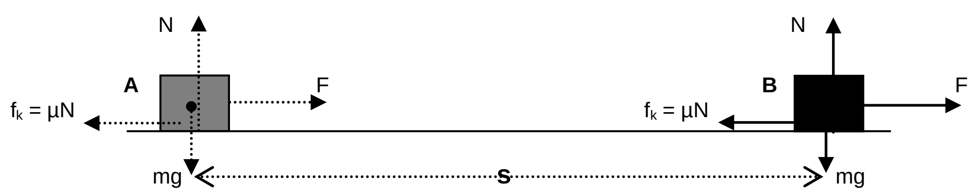
Similarly if we change the angle of inclination from θ to θ_1, then work done will be

$$\mathbf{W_1 = mgh + \frac{\mu mgh}{Tan\theta_1}}$$

This clearly shows that work done in both the cases is different & hence work done against non-conservative force in a path function and not a state function i.e. it not only depends upon initial & final states of body but also depends upon the path through which process has been carried out.

To Prove That Work Done Against Non-conservative Forces In A Complete Cycle Is Not Zero

Consider a body displaced slowly on a rough horizontal plane through displacement s from A to B.



Minimum force required to move the body is

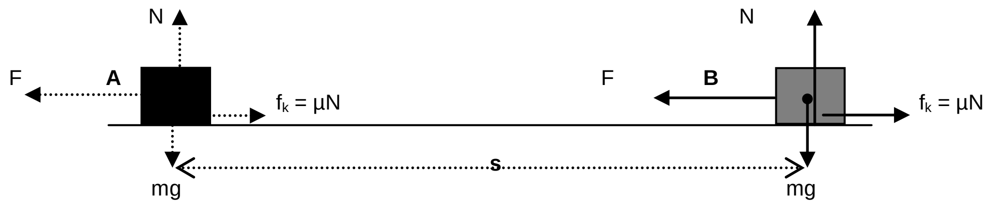
$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0$$ (since force and displacement are in same direction)

or, $$\mathbf{W = \mu mgs}$$

Now if the same body is returned back from B to A



Minimum force required to move the body is

$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0$$ (since force and displacement are in same direction)

or, $$\mathbf{W = \mu mgs}$$

Hence total work done in the complete process

$$\mathbf{W = W_1 + W_2 = 2\mu mgs}$$

Note - When body is returned from B to A friction reverse its direction.

POWER

Rate of doing work by a body with respect to time is known as power.

Average Power

It is defined as the ratio of total work done by the body to total time taken.

$$\mathbf{P_{avg} = \frac{Total\ work\ done}{Total\ time\ taken} = \frac{\Delta W}{\Delta t}}$$

Instantaneous Power

Power developed within the body at any particular instant of time is known as instantaneous power.

Or

Average power evaluated for very short duration of time is known as instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} P_{avg}$$

or, $$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$P_{inst} = \frac{dW}{dt}$$

or, $$P_{inst} = \frac{d\vec{F} \cdot \vec{s}}{dt}$$

or, $$P_{inst} = \vec{F} \cdot \frac{d\vec{s}}{dt}$$

or, $$\mathbf{P_{inst} = \vec{F} \cdot \vec{v}}$$

EFFICIENCY

It is defined as the ratio of power output to power input.

Or

It is defined as the ratio of energy output to energy input.

Or

I It is defined as the ratio of work output to work input.

$$\eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}}$$

PERCENTAGE EFFICIENCY

Percentage Efficiency = Efficiency x 100

$$\textbf{Percentage Efficiency} = \eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}} \times 100$$

COLLISION

Collision between the two bodies is defined as mutual interaction of the bodies for a short interval of time due to which the energy and the momentum of the interacting bodies change.

Types of Collision

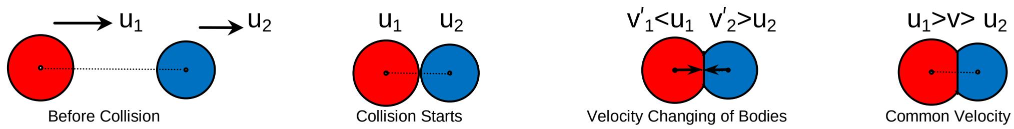
There are basically three types of collisions-

i) Elastic Collision – That is the collision between perfectly elastic bodies. In this type of collision, since only conservative forces are operating between the interacting bodies, both kinetic energy and momentum of the system remains constant.

ii) Inelastic Collision – That is the collision between perfectly inelastic or plastic bodies. After collision bodies stick together and move with some common velocity. In this type of collision only momentum is conserved. Kinetic energy is not conserved due to the presence of non-conservative forces between the interacting bodies.

iii) Partially Elastic or Partially Inelastic Collision – That is the collision between the partially elastic bodies. In this type of collision bodies do separate from each other after collision but due to the involvement of non-conservative inelastic forces kinetic energy of the system is not conserved and only momentum is conserved.

Collision In One Dimension – Analytical Treatment



Consider two bodies of masses m_1 and m_2 with their center of masses moving along the same straight line in same direction with initial velocities u_1 and u_2 with m_1 after m_2. Condition necessary for the collision is $u_1 > u_2$ due to which bodies start approaching towards each other with the velocity of approach $u_1 - u_2$.
Collision starts as soon as the bodies come in contact. Due to its greater velocity and inertia m_1 continues to push m_2 in the forward direction whereas m_2 due to its small velocity and inertia pushes m_1 in the backward direction. Due to this pushing force involved between the two colliding bodies they get deformed at the point of contact and a part of their kinetic energy gets consumed in the deformation of the bodies. Also m_1 being pushed opposite to the direction of the motion goes on decreasing its velocity and m_2 being pushed in the direction of motion continues increasing its velocity. This process continues until both the bodies acquire the same common velocity v. Up to this stage there is maximum deformation in the bodies maximum part of their kinetic energy gets consumed in their deformation.

Elastic collision

In case of elastic collision bodies are perfectly elastic. Hence after their maximum deformation they have tendency to regain their original shapes, due to which they start pushing each other. Since m_2 is being pushed in the direction of motion its velocity goes on increasing and m_1 being pushed opposite to the direction of motion its velocity goes on decreasing. Thus condition necessary for separation i.e. $\mathbf{v_2 > v_1}$ is attained and the bodies get separated with velocity of separation $\mathbf{v_2 - v_1}$.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is again returned back to the system when the bodies regain their original shapes. Hence in such collision energy conservation can also be applied along with the momentum conservation.

Applying energy conservation

$$E_i = E_f$$
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$
$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ ....(i)}$$

Applying momentum conservation

$$p_i = p_f$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ....(ii)}$$

Dividing equation (i) by (ii)

$$u_1 + v_1 = v_2 + u_2$$

or, $$\mathbf{v_2 - v_1 = u_1 - u_2}$$

or, **Velocity of separation = Velocity of approach**

or, $$v_2 = v_1 + u_1 - u_2$$

Putting this in equation (i)

$$\mathbf{v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2 u_2}{(m_1 + m_2)}}$$

Similarly we can prove

$$\mathbf{v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1 u_1}{(m_1 + m_2)}}$$

Case 1- If the bodies are of same mass,

$$m_1 = m_2 = m$$

$$\mathbf{v_1 = u_2}$$

$$\mathbf{v_2 = u_1}$$

Hence in perfectly elastic collision between two bodies of same mass, the velocities interchange.ie. If a moving body elastically collides with a similar body at rest. Then the moving body comes at rest and the body at rest starts moving with the velocity of the moving body.

Case 2- If a huge body elastically collides with the small body,

$m_1 >> m_2$

m_2 will be neglected in comparison to m_1

$$v_1 = \frac{(m_1 - 0)u_1}{(m_1 + 0)} + \frac{2.0.\, u_2}{(m_1 + 0)}$$

$$\mathbf{v_1 = u_1}$$

and

$$v_2 = \frac{(0 - m_1)u_2}{(m_1 + 0)} + \frac{2m_1 u_1}{(m_1 + 0)}$$

$$\mathbf{v_2 = -u_2 + 2u_1}$$

If, $u_2 = 0$

$$\mathbf{v_2 = 2u_1}$$

Hence if a huge body elastically collides with a small body then there is almost no change in the velocity of the huge body but if the small body is initially at rest it gets thrown away with twice the velocity of the huge moving body.eg. collision of truck with a drum.

Case 3- If a small body elastically collides with a huge body,

$m_2 >> m_1$

m_1 will be neglected in comparison to m_2

$$v_1 = \frac{(0 - m_2)u_1}{(0 + m_2)} + \frac{2m_2 u_2}{(0 + m_2)}$$

or, $\mathbf{v_1 = -u_1 + 2u_2}$

If $u_2 = 0$

$$\mathbf{v_1 = -u_1}$$

and

$$v_2 = \frac{(m_2-0)u_2}{(0+m_2)} + \frac{2.0.u_1}{(0+m_2)}$$

$$\mathbf{v_2 = u_2}$$

Hence if a small body elastically collides with a huge body at rest then there is almost no change in the velocity of the huge body but if the huge body is initially at rest small body rebounds back with the same speed.eg. collision of a ball with a wall.

Inelastic collision

In case of inelastic collision bodies are perfectly inelastic. Hence after their maximum deformation they have no tendency to regain their original shapes, due to which they continue moving with the same common velocity.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is permanently consumed in the deformation of the bodies against non-conservative inelastic forces. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

Applying momentum conservation

$$p_i = p_f$$

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

or, $m_1u_1 + m_2u_2 = (m_1+m_2)v$

or, $$\mathbf{v = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)}}$$

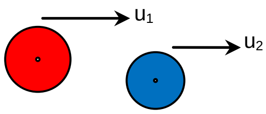
Partially Elastic or Partially Inelastic Collision

In this case bodies are partially elastic. Hence after their maximum deformation they have tendency to regain their original shapes but not as much as perfectly elastic bodies. Hence they do separate but their velocity of separation is not as much as in the case of perfectly elastic bodies i.e. their velocity of separation is less than the velocity of approach.

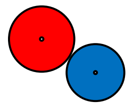
In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is only slightly returned back to the system. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

$$\mathbf{(v_2 - v_1) < (u_1 - u_2)}$$

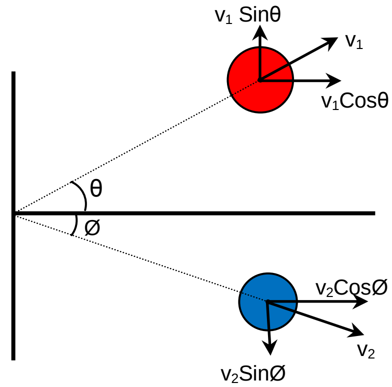
Collision In Two Dimension – Oblique Collision



Before Collision



Collision Starts



After Collision

When the centers of mass of two bodies are not along the same straight line, the collision is said to be oblique. In such condition after collision bodies are deflected at some angle with the initial direction. In this type of collision momentum conservation is applied separately along x-axis and y-axis. If the collision is perfectly elastic energy conservation is also applied.

Let initial velocities of the masses m_1 and m_2 be u_1 and u_2 respectively along x-axis. After collision they are deflected at angles θ and Ø respectively from x-axis, on its either side of the x axis.

Applying momentum conservation along x-axis

$$p_f = p_i$$

$$\mathbf{m_1 v_1 \cos\theta + m_2 v_2 \cos Ø = m_1 u_1 + m_2 u_2}$$

Applying momentum conservation along y-axis

$$p_f = p_i$$

$$m_1 v_1 \sin\theta - m_2 v_2 \sin Ø = m_1 0 + m_2 0$$

or, $$m_1 v_1 \sin\theta - m_2 v_2 \sin Ø = 0$$

or, $$\mathbf{m_1 v_1 \sin\theta = m_2 v_2 \sin Ø}$$

In case of elastic collision applying energy conservation can also be applied

$$K_f = K_i$$

$$\mathbf{\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$$

Coefficient Of Restitution

It is defined as the ratio of velocity of separation to the velocity of approach.

$$\mathbf{e = \frac{Velocity\ of\ separation}{Velocity\ of\ approach}}$$

or, $$e = \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

Case-1 For perfectly elastic collision, velocity of separation is equal to velocity of approach, therefore

$$e = 1$$

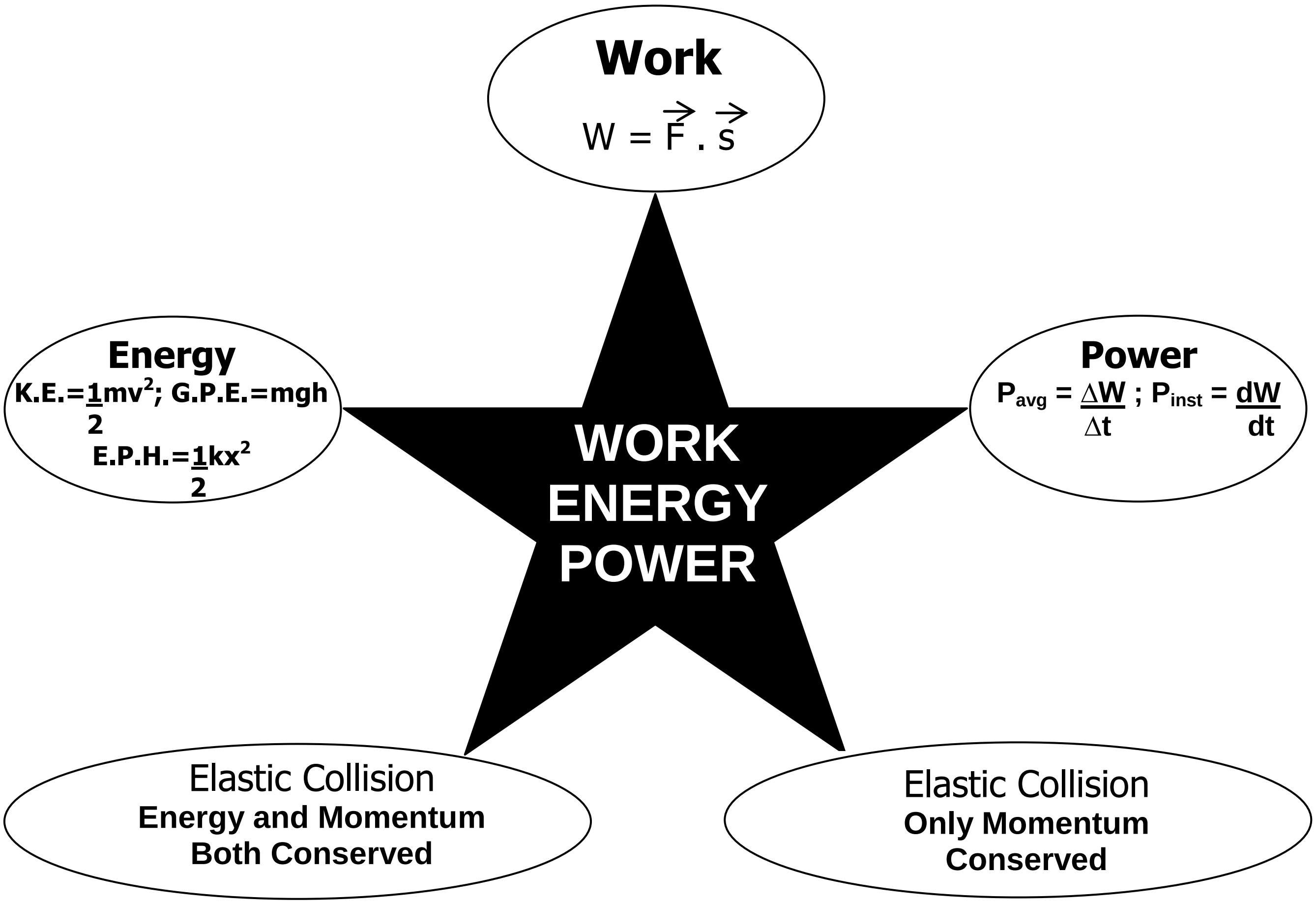
Case-2 For perfectly inelastic collision, velocity of separation is zero, therefore

$$e = 0$$

Case-3 For partially elastic or partially inelastic collision, velocity of separation is less than velocity of approach, therefore

$$e < 1$$

MEMORY MAP


Work
$W = \vec{F} \cdot \vec{s}$
Energy
K.E.$=\frac{1}{2}mv^2$; G.P.E.=mgh
E.P.H.$=\frac{1}{2}kx^2$
Power
$P_{avg} = \frac{\Delta W}{\Delta t}$; $P_{inst} = \frac{dW}{dt}$
WORK
ENERGY
POWER
Elastic Collision
Energy and Momentum
Both Conserved
Elastic Collision
Only Momentum
Conserved

Very Short Answer Type 1 Mark Questions

1. Define the conservative and non-conservative forces? Give example of each?

2. A light body and a heavy body have same linear momentum. Which one has greater K.E? (Ans: Lighter body has more K.E.)

3.If the momentum of the body is doubled by what percentage does its K.E changes? (300%)

4. A truck and a car are moving with the same K.E on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance? (Truck)

5. What happens to the P.E of a bubble when it rises up in water? (decrease)

6. Define spring constant of a spring?

7. What happens when a sphere collides head on elastically with a sphere of same mass initially at rest?

8. Derive an expression for K.E of a body of mass m moving with a velocity v by calculus method.

9. After bullet is fired, gun recoils. Compare the K.E. of bullet and the gun. (K.E. of bullet > K.E. of gun)

10. In which type of collision there is maximum loss of energy?

Very Short Answer Type 2 Marks Questions

1. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10% what is the speed with which the bob arrives the lowest point? (**Ans :** 6m/s)

2. Find the work done if a particle moves from position $\vec{r_1}$ = (4i + 3j + 6k)m to a position $\vec{r_2}$ = (14i = 13j = 16k) under the effect of force, $\vec{F}$ = (4i + 4j - 4k)N? (**Ans :** 40J)

3. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is stretched further through 0.1m calculate work done? (**Ans :** 4000 Nm–1, 60 J)

4. A pump on the ground floor of a building can pump up water to fill a tank of volume $30m^3$ in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump? The efficiency of the pump is 30%. (**Ans :** 43.556 kW)

5. Spring of a weighing machine is compressed by 1cm when a sand bag of mass 0.1 kg is dropped on it from a height 0.25m. From what height should the sand bag be dropped to cause a compression of 4cm? (**Ans :** 4m)

6. Show that in an elastic one dimensional collision the velocity of approach before collision is equal to velocity of separation after collision?

7. A spring is stretched by distance x by applying a force F. What will be the new force required to stretch the spring by 3x? Calculate the work done in increasing the extension?

8. Write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by length. Draw the graphs to show the variation of potential energy and force with elongation?

9. How does a perfectly inelastic collision differ from perfectly elastic collision? Two particles of mass m_1 and m_2 having velocities u_1 and u_2 respectively make a head on collision. Derive the relation for their final velocities?

10. In lifting a 10 kg weight to a height of 2m, 250 Joule of energy is spent. Calculate the acceleration with which it was raised?(g=10m/s^2) (**Ans :** 2.5m/s^2)

Short Answer Type 3 Marks Questions

1. An electrical water pump of 80% efficiency is used to lift water up to a height of 10m.Find mass of water which it could lift in 1hrour if the marked power was 500 watt?
2. A cycle is moving up the incline rising 1 in 100 with a const. velocity of 5m/sec. Find the instantaneous power developed by the cycle?
3. Find % change in K.E of body when its momentum is increased by 50%.
4. A light string passing over a light frictionless pulley is holding masses m and 2m at its either end. Find the velocity attained by the masses after 2 seconds.
5. Derive an expression for the centripetal force experienced by a body performing uniform circular motion.
6. Find the elevation of the outer tracks with respect to inner. So that the train could safely pass through the turn of radius 1km with a speed of 36km/hr. Separation between the tracks is 1.5m?
7. A block of mass m is placed over a smooth wedge of inclination θ. With what horizontal acceleration the wedge should be moved so that the block must remain stationery over it?
8. Involving friction prove that pulling is easier than pushing if both are done at the same angle.
9. In vertical circular motion if velocity at the lowermost point is $\sqrt{(6rg)}$ where find the tension in the string where speed is minimum. Given that mass of the block attached to it is m?
10. A bullet of mass m moving with velocity u penetrates a wooden block of mass M suspended through a string from rigid support and comes to rest inside it. If length of the string is L find the angular deflection of the string.

Long Answer Type 5 Marks Questions

1. What is conservative force? Show that work done against conservative forces is a state function and not a path function. Also show that work done against it in a complete cycle is zero?
2. A body of man 10 kg moving with the velocity of 10m/s impinges the horizontal spring of spring constant 100 Nm^{-1} fixed at one end. Find the maximum compression of the spring? Which type of mechanical energy conversion has occurred? How does the answer in the first part changes when the body is moving on a rough surface?
3. Two blocks of different masses are attached to the two ends of a light string passing over the frictionless and light pully. Prove that the potential energy of the bodies lost during the motion of the blocks is equal to the gain in their kinetic energies?

4. A locomotive of mass m starts moving so that its velocity v is changing according to the law v $\sqrt{(as)}$, where a is constant and s is distance covered. Find the total work done by all the forces acting the locomotive during the first t seconds after the beginning of motion?

5. Derive an expression for the elastic potential energy of the stretched spring of spring constant k. Find the % change in the elastic potential energy of spring if its length is increased by 10%?

Some Intellectual Stuff

1. A body of mass m is placed on a rough horizontal surface having coefficient of static friction μ with the body. Find the minimum force that must be applied on the body so that it may start moving? Find the work done by this force in the horizontal displacement s of the body?

2. Two blocks of same mass m are placed on a smooth horizontal surface with a spring of constant k attached between them. If one of the block is imparted a horizontal velocity v by an impulsive force, find the maximum compression of the spring?

3. A block of mass M is supported against a vertical wall by a spring of constant k. A bullet of mass m moving with horizontal velocity v_0 gets embedded in the block and pushes it against the wall. Find the maximum compression of the spring?

4. Prove that in case of oblique elastic collision of a moving body with a similar body at rest, the two bodies move off perpendicularly after collision?

5. A chain of length L and mass M rests over a sphere of radius R (L < R) with its one end fixed at the top of the sphere. Find the gravitational potential energy of the chain considering the center of the sphere as the zero level of the gravitational potential energy?